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S. Gutman

Root Clustering in Parameter Space

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Author
Prof. Shaul Gutman
Dept. of Mechanical Engineering
Technion – Israel Institute of Technology
Haifa, Israel

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To my mother

אשת חייל מי ימצא

PREFACE

In 1763 Waring discovered that aperiodicity can be tested using the minus square of the root-pair-difference. This discovery marked the beginning of root clustering investigation. The important ideas of stability and later relative stability, introduced in the 19th century, motivated the following general problem: to find a criterion for the inclusion of the eigenvalues of a given matrix in a prescribed algebraic region in the complex plane. The extensive research carried out in recent years, has brought the subject to a reasonable level of maturity; thus I feel it is about time to bridge the 220 years with an appropriate book. This, to the best of my knowledge, is the first book dealing with the general root clustering problem.

The book covers five basic topics: First - a review of classical results; second - root clustering criteria based on one variable transformation; third - criteria based on composite matrices and polynomials; fourth - criteria based on linear matrix equations; fifth - the image of the criteria in the parameter space, including an application to feedback.

This book should prove valuable for systems and control engineers as well as for mathematicians. In other areas such as physics, the results may be of help in the analysis of stability properties.

Although the book reflects my personal view of the subject, I have included, for completeness, other main approaches to root clustering. The theme of the book is general structures of root clustering criteria, and their image in the parameter space. It is not my intention, however, to replace Routh table, for instance, by a more complicated criterion.

My interest in the subject originated in the pleasant atmosphere of Berkeley, more than a decade ago. I soon realized that there is no contradiction in working with Professor George Leitmann on differential games, min-max, etc., while at the same time being involved with Professor Eli Jury in new ideas about stability. George's devotion to mathematical carefulness and Eli's enthusiasm for stability contributed much to my education. However, most of my research on root clustering has developed during my work at the Technion, thanks to the the friendly spirit of my colleagues in the Department of Mechanical Engineering. Among my past students I wish to mention Dr. Fabien Chojnowski and Dr. Hedi Taub who made a significant contribution to the theory presented in the book. Part of the results in Sections 7.5, 7.6 , and the Appendix are due to my Ph.D. student Mani Fischer. I wish to thank Mrs. R. Alon and Mrs. M. Schreier for typing the manuscript and I. Katner for computer drawings. Last but not least, I wish to thank my wife Yaffa and my children, Rakefet, Oren and Michal who inspired my life and work. Without their patience and support, this work would not have been completed.

תם ונשלם שבח לאל בורא עולם

תשרי התש״ן

Shaul Gutman

Haifa, Israel, Oct. 1989

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LIST OF SYMBOLS

ℵ	a region in the complex plane
cl(ℵ)	the closure of ℵ
$\mathbf{R}$, R	the set of real numbers
$\mathbf{C}$, C	the set of complex numbers
$\mathbf{R}^n$, R^n	n-dimensional real space
$\mathbf{C}^n$, R^n	n-dimensional complex space
$\mathbf{R}[z_1, ..., z_n]$	the set of n-variate real polynomials
$\mathbf{C}[z_1, ..., z_n]$	the set of n-variate complex polynomials
$\times$	Cartesian product
$\in$	a member of
$\cup$	union
$\subset$	is a subject of
$\cap$	intersection
$\forall$	for all
$\exists$	there exists
$\varnothing$	the empty set
$\Leftrightarrow$	if and only if
$\mathbf{R}^{n\times m}$, $R^{n\times m}$	the set of n×m real matrices
$\mathbf{C}^{n\times m}$, $C^{n\times m}$	the set of n×m complex matrices
$\bar{a}$	complex conjugate of a
A'	matrix transpose
A^*	matrix conjugate transpose
$\otimes$	Kronecker product
$\odot$	bialternate product
$\cdot$	Schur (term by term) product
$\downarrow$	stacking operator
$\uparrow$	inverse stacking operator
tr	trace of a matrix
σ	spectrum, collection of all eigenvalues (roots)
det	determinant of a matrix
$\vert\ \vert$	absolute value, determinant
$\Vert\ \Vert$	norm
p.d.	positive definite
p.s.d.	positive semidefinite
Coef	all coefficients of a polynomial
$\overset{\text{mod}}{=}$	modular equality; equals on the spectrum
Min	minimum
s.t.	subject to