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Flavio Dobran

Theory of Structured Multiphase Mixtures



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Preface

The purpose of this monograph is to present a theory of multiphase mixtures with structure. The development of the theory is guided by physical principles without the sacrifice of mathematical rigor. The starting point in the development of the theory is based on the author's previous work, utilizing the volume averaging approach to motivate the construction of the theory. In the monograph, this basic idea is exploited further to introduce an additional set of transport equations for the purpose of modeling the structural properties of the mixture. As such, much of the material presented in the book is new and it should be useful for further work by graduate students and applied mathematicians and scientists. These studies may involve constitutive equations, wave propagation, numerical solution of partial differential equations, turbulence, and extensions of the theory to include higher order material deformations.

The prerequisite to understanding the mathematics in this book requires an elementary knowledge of algebra, geometry, and calculus. It should be readily accessible to students having backgrounds in fluid mechanics and thermodynamics in which they have been exposed to the cartesian tensor analysis. To make the book selfcontained, an appendix is provided in which the basic mathematical notions of algebra, geometry, and calculus are reviewed. The employed notation is by no means established in the field, but the one chosen has been extensively used in the literature of the single phase multicomponent mixture theories and many workers in this field are familiar with it.

The book is divided into eight chapters. Chapter one summarizes important historical milestones leading to the development of structured models of multiphase mixtures. In particular, the need for reconciliation between different approaches in modeling multiphase mixtures and the inclusion of structural characteristics of mixtures within the theory are discussed and a set of objectives for the book are stated. The development of the theory is carried out in chapter two, whereas in chapter three this development is continued on the basis of a very general principle of the material deformation with respect to the center of mass of each phase. The restriction of the resulting set of field equations by the principle of material frame-indifference is discussed in chapter four. The special cases of the field equations are compared with the existing models in chapter five. Chapter six includes the development of concepts and principles of the constitutive theory which is used in chapters seven and eight to study constitutive

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equations. The latter chapters include discussions of special results for two-phase mixtures involving compressible and incompressible phases, and concentrated and dilute suspensions with and without negligible inertial effects.

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