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Numerical Operations with Polynomial Matrices

Application to Multi-Variable Dynamic
Compensator Design



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