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Roland Klees Roger Haagmans (Eds.)

Wavelets in the Geosciences

With 102 Figures and 3 Tables

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Preface

This book is a collection of the Lecture Notes of the School of Wavelets in the Geosciences held in Delft (The Netherlands) from 4-9 October 1998. The objective of the school was to provide the necessary information for understanding the potential and limitations of the application of wavelets in the geosciences. Three lectures were given by outstanding scientists in the field of wavelet theory and applications: Dr. Matthias Holschneider, Laboratoire de Geomagnetisme, Institut de Physique du Globe De Paris, France; Dr. Wim Sweldens, Mathematical Sciences Research Centre, Lucent Technologies, Bell Laboratories, Murray Hill, NJ, U.S.A.; Prof. Dr. Willi Freeden, Geomathematics Group, Department of Mathematics, University of Kaiserslautern, Germany. The lectures have been supplemented by intensive computer exercises.

The school has been very successful due to the engagement and the excellent presentations of the teachers, the very illustrative and instructive computer exercises, the lively interest and participation of the participants, and the many fruitful discussions we had. Therefore I want to express my thanks to the teachers for the excellent job they did, for providing typewritten Lecture Notes, and for their excellent co-operation. Thanks also to the students, who were actively engaged in the lectures and exercises during the whole week.

The organisation of such a School is not possible without the support of many others. First of all I want to thank Prof. Dr. Willi Freeden and his co-workers from the Geomathematics Group, Department of Mathematics, University of Kaiserslautern, who co-organised the school. They provided a considerable contribution to the success of the school.

I also want to express my thanks to the support of Michael Bayer, Martin van Gelderen, Roger Haagmans and my secretary, Wil Coops-Luyten, who were heavily involved in the organisation of the school and in all practical aspects including the wonderful social program which gave the participants some flavour of Dutch culture and the beautiful city of Delft. In the work of organisation, we have been supported by all the staff of Section Physical, Geometric and Space Geodesy (FMR). Last but not least I want to thank Martin van Gelderen and Roger Haagmans for preparing the introduction.

Essential to the success of the school was the support we received from various organisations: the International Association of Geodesy (IAG), the Netherlands Geodetic Commission (NCG), the Department of Geodesy at Delft University of Technology, and the Delft Institute for Earth-Oriented Space Research (DEOS). Their support is gratefully acknowledged.

Delft, October 1999 Roland Klees

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Introduction

Background

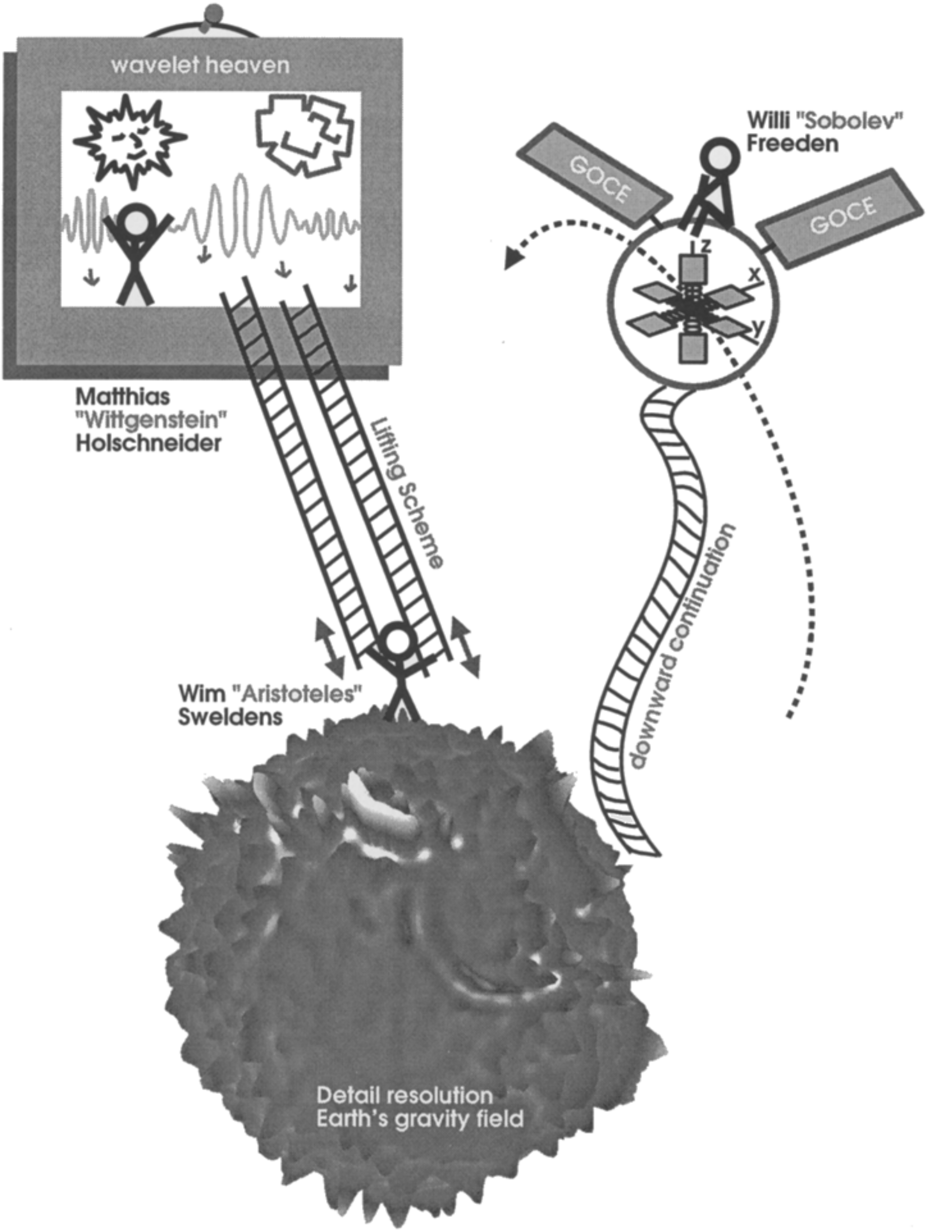
The international school on Wavelets in the Geosciences fits within the tradition of the summer schools of the International Association of Geodesy that exist already since many years. The topics are generally carefully selected and cover an interesting field of current and future interest. In this respect the wavelets fulfill such expectations; they offer challenging mathematical opportunities and many fields of application. The range of applications is so wide that the focus was limited to the geosciences. In this way participants with different background were offered the opportunity to meet and exchange ideas with each other and with an excellent team of lecturers, consisting of Matthias Holschneider, Wim Sweldens and Willi Freeden. The school was organized by prof. R. Klees of the Delft University of Technology in the Netherlands and by prof. W. Freeden of the University of Kaiserslautern in Germany. The school was hosted by the Department of Geodesy of the Faculty of Civil Engineering and Geosciences of the Delft University of Technology in Delft, the Netherlands. It was held from October 4th to October 9th 1998. The number of participants was limited to 41 due to the restricted number of computers available for the exercises. The group was divers in several respects: 5 continents and 19 countries were represented. The majority of participants were Ph.D. students or members of the academic staff, and the group was completed by a few post-docs, M.Sc. students and non-academics. Around 29% had a background in geodesy, 48% in geophysics, 19% in mathematics and the rest in other disciplines. So, from the point of view of bringing people from different background and different nationalities together in a stimulating scientific environment, which is one of the major goals of the "summer" schools, it was a success.

The basic objective of the school was to provide the necessary information to understand the potential and limitations of the application of wavelets in the geosciences. This includes:

- the mathematical representation in one and more dimensions like on the sphere
- the properties as compared to Fourier techniques
- the signal representation and analysis ability
- the use of operators in terms of wavelets
- gaining experiences with wavelets using examples from geosciences in computer exercises

Lectures and Notes

The school lasted six days and contained three major subjects. Every subject was covered in two days time. All topics were supported by practical exercises on the



Artist's impression of the school

computer with examples from geodynamics, topography representation, gravity field modeling etc. The program consisted of three parts that are described in these lecture notes. The idea behind the total program is to present the current status of continuous wavelet analysis for data analysis applications in the geosciences as a first step. The second part has a more specific focus on discrete analysis with special emphasis on the second generation wavelets. This allows to loosen the link with Fourier analysis, and the extension to complex surfaces and fast and efficient data approximation. The third part presents the current status of global data analysis on the sphere with the potential field as a special field of application. This setup makes it possible for participants and readers to get a view on wavelets also beyond their own field of interest which may lead to curiosity, discussion and possibly new developments.

Part 1

The first part was lectured and written by Matthias Holschneider. The basic facts about harmonic analysis through Fourier transforms in one dimension are recalled. In particular several sampling theorems are presented linking the Fourier transform over the real line to the Fourier transform over the circle and to the so called FFT (Fourier transform over the discretize circle). This is important in applications since only the FFT is implementable on computers, however the signal one treats are at least in spirit functions over the real line. Many "errors" in data processing come from a non clear distinction between the various transforms.

To remedy with certain shortcomings of standard Fourier techniques the wavelet transform is introduced as a possible time-frequency technique. Important general features of wavelet transform are discussed such as energy conservation, inversion formulas, covariance properties and reproducing kernels. This is important to be able to read wavelet transforms of signals. Also wavelet transforms of functions on the circle (periodic functions) are treated. The possibility of down-sampling of wavelet coefficients is linked to the existence of frames and orthonormal bases of wavelets.

Two algorithms for the implementation of continuous wavelet transform are presented. The first is based on the computation of convolutions using the FFT with a sorrow boundary treatment. The second is an algorithm based on a dyadic interpolation scheme.

Several applications in the field of non stationary filtering, singularity processing, and detection of sources of potential fields are evaluated and discussed. First, upon manipulating and modifying wavelet coefficients rather than Fourier coefficients, it is shown how to construct non stationary filters. In examples this technique is applied to the decomposition of the polar motion of the earth into several components. The underlying algebra of wavelet Toeplitz operators is discussed. In particular the user is made aware of problems of non-commutativity. Wavelet transforms of noise and de-noising methods are treated based on adaptive filter design in wavelet space (thresholding). Secondly, it is shown how local singularities may be detected and processed through wavelet techniques. In particular how the scaling behavior of wavelet coefficients reveals the local singularity exponent. Applications of this to fractal analysis of data (extraction of generalized fractal wavelet dimension) are introduced. In particular the wavelet correlation dimension is important for the interpretation of scattering on fractal objects. In the case of isolated singularities the wavelet technique is applied to the detection analysis of geomagnetic jerks in the data of secular variation of the magnetic field. Thirdly, a special family of wavelets based on the Poisson semi-group is used to detect hidden singularities, corresponding to remote sources of potential fields. Only the remote field is available for the analysis, but cross scale relations of the wavelet coefficients may be used to localize and characterize sources of the field inhomogeneity.

Part 2

The second part was lectured by Wim Sweldens and the lecture notes are co-authored by Peter Schröder, and Ingrid Daubechies.

Starting from historical developments they try to answer the question: Why multi-resolution? In order to illustrate this the contrast with Fourier methods is explained, and the connection with filter banks is shown. This is illustrated with a simple example: Haar wavelets.

At this point the focus is on the need for spatial constructions, as opposed to Fourier based constructions, revisiting Haar. Here, the essentials of lifting are introduced: predict and update. The next step goes beyond Haar, where linear and higher order polynomial based wavelet are considered. Finally, implementation aspects of lifting, such as speeding up, in-place memory calculations, and integer to integer transforms get special attention.

The lecture proceeds with a look at the need for second generation wavelets. Aspects of importance are wavelets for boundaries, domains, irregular samples, weighted measures, and manifolds. Here the use of lifting in order to construct second generation wavelets is explained.

In order to construct wavelets on the sphere one needs to build triangular grids on a sphere: geodesic sphere construction. Also the definition of multi-resolution on a sphere needs to be defined. Then one can focus on constructing filter operators on a sphere. Again lifting is used now to build spherical wavelets. This is where one arrives at the fast spherical wavelet transform. An application is discussed from spherical image processing.

The next discussion points at techniques for triangulating general 2-manifolds. For this purpose wavelets defined on 2-manifolds are required, and multi-resolution transforms "off" manifolds are introduced. This is supported by examples.

Part 3

The third and last part of the school is lectured by Willi Freeden and the lecture notes are co-authored by Volker Michel with a focus on harmonic wavelets on the sphere.

For the determination of the earth's gravity field many types of observations are nowadays available, such as terrestrial gravimetry, airborne gravimetry, satellite to satellite tracking, satellite gradiometry. The mathematical connection between these observables on the one hand and the gravity field and the shape of the earth on the other is called the integrated concept. In this lecture windowed Fourier transforms and harmonic wavelets on the sphere are introduced for example for approximating the gravitational part of the gravity field progressively better and better.

The classical outer harmonic models of physical geodesy, concerned with the earth's gravity field, triggered the development of the integrated concept in terms of bounded linear functionals on reproducing Hilbert (Sobolev) spaces. It is of importance to deal with completeness properties and closure theorems for dense systems of linear (observational) functionals acting on outer harmonics and reproducing kernel functions.

The uncertainty principle for functions harmonic outside a sphere is treated. This leads us to the terminology of 'space-frequency localization'. It is shown that the uncertainty principle is a restriction in gravitational field determination which tells us that sharp localization in space and frequency is mutually exclusive. Finally, a space-frequency investigation is considered for the most important trial functions used in physical geodesy.

Here two types of scaling functions can be distinguished, viz. band-limited and non-band-limited. It is illustrated that in all cases the constituting elements of a multilevel approximation by convolution consist of 'dilation' and 'shifting' of a mother kernel, i.e. a potential with vanishing zero order moment. Next, the concept of multi-resolution analysis for Sobolev spaces of harmonic functions is introduced which is especially relevant for geophysical purposes.

Two possible substitutes of the Fourier transform in geopotential determination are the Windowed Fourier Transform (WFT) and the Wavelet Transform (WT). The harmonic WFT and WT are introduced and it is shown how these can be used to give information about the geopotential simultaneously in the space domain and the frequency (angular momentum) domain. The counterparts of the inverse Fourier transform are derived, which allows a reconstruction of the geopotential from its WFT and WT, respectively. Moreover, a necessary and sufficient condition is derived, that an otherwise arbitrary function of space and frequency has to satisfy in order to be the WFT or WT of a potential. Finally, least-squares approximation and minimum norm (i.e. least-energy) representation, which will play a particular role in geodetic applications of both WFT and WT, are discussed in more detail.

Exercises and Demonstrations

All lectures were accompanied by practical exercises using flexible wavelet programs. Academic signals as well as true geophysical data could be treated and analysed. Next to these 'hands-on' sessions also examples from a diversity of more complex or computationally intensive applications to support the theory were demonstrated. Two videos illustrated the power of the methods in the field of computer animation and earth gravity field approximation. The enthusiasm of the lecturers and the combination of theoretical foundations and developments, and the link to the practical applications lead to a good insight into the current status and the future challenges in the field of wavelets in the geosciences.

We very much enjoyed organizing and taking part in the school so we hope that you enjoy reading the book and applying the wavelets in a similar fashion.

Delft, October 1999

Roger Haagmans
Martin van Gelderen