

Lecture Notes in Physics

Edited by H. Araki, Kyoto, J. Ehlers, München, K. Hepp, Zürich
R. Kippenhahn, München, D. Ruelle, Bures-sur-Yvette
H. A. Weidenmüller, Heidelberg, J. Wess, Karlsruhe and J. Zittartz, Köln
Managing Editor: W. Beiglböck

335

A. Lakhtakia
V. K. Varadan
V. V. Varadan

Time-Harmonic
Electromagnetic Fields
in Chiral Media



Springer-Verlag

Berlin Heidelberg New York London Paris Tokyo Hong Kong

Authors

A. Lakhtakia

V.K. Varadan

V.V. Varadan

Department of Engineering Science & Mechanics
and Research Center for the Engineering of
Electronic and Acoustic Materials
227 Hammond Building, Pennsylvania State University
University Park, PA 16802, USA

ISBN 3-540-51317-5 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-51317-5 Springer-Verlag New York Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. Duplication of this publication or parts thereof is only permitted under the provisions of the German Copyright Law of September 9, 1965, in its version of June 24, 1985, and a copyright fee must always be paid. Violations fall under the prosecution act of the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1989
Printed in Germany

Printing: Druckhaus Beltz, Hemsbach/Bergstr.
Binding: J. Schäffer GmbH & Co. KG., Grünstadt
2158/3140-543210 – Printed on acid-free paper

Frontispiece: Maori drawing of a spiral facial tattoo [After R. Huber, *Treasury of Fantastic and Mythological Treasures*, New York: Dover (1981)]



PREFACE

With ongoing progress in the construction of artificial composites, chiral materials will assume great importance. Such materials occur in nature as optically active molecules which display circular birefringence at optical frequencies. Since chirality is a geometric concept, it is conceivable that artificially chiral composites can be made to display circular birefringence even at lower, perhaps microwave, frequencies.

These lecture notes have been prepared to initiate practitioners of electromagnetic theory into the arcana of chiral media. The notes are intended to cover twenty class periods, and have been accordingly arranged. Though Sections 1 through 19 focus exclusively on electromagnetic fields, an extension of these concepts to acoustic waves is given in Section 20 in order to emphasize that it is geometry which gives rise to chirality. An $\exp[-i\omega t]$ harmonic time dependence is implicit throughout this work; only in Section 6 will time-dependent fields be considered. Familiarity with basic electromagnetic theory at the graduate level is assumed, and some experience with dyadic algebra will be useful but not necessary. As such, these notes are suitable for advanced graduate students as well as for researchers with background in electromagnetic theory. We trust that these notes will inspire a considerable number of researchers to be as fascinated by chiral media as we are.

A note on the notation used here: Generally, new notation has been explained upon introduction. An $\exp[-i\omega t]$ harmonic time dependence is implicit throughout, the sole exception being in Section 6. Boldface letters have been used to denote vectors, while German letters denote dyadics. The symbols $\{\mathbf{E}, \mathbf{H}, \mathbf{D}, \mathbf{B}\}$ have been used consistently to denote complex-valued field amplitudes; in the first part of Section 6, however, these symbols denote complex instantaneous fields. The symbol σ has been used as a wavenumber in defining wavefunctions, such as $\mathbf{M}_n^{(1)}(\sigma\mathbf{r})$ and $\mathbf{N}_n^{(1)}(\sigma\mathbf{r})$ of Section 10, as well as the scalar Green's function $g(\sigma;R) = \exp[i\sigma R]/R$. The wavenumbers in an isotropic chiral medium are denoted by γ_1 for the left-circularly polarized fields, and by γ_2 for the right-circularly polarized fields; it is to be emphasized here that $k = \omega(\epsilon\mu)^{1/2}$ is not a wavenumber in a chiral medium. Throughout this work, it has been assumed that the chiral media are homogeneous as well as at rest. Special notation has been used for acoustically chiral solids in Section 20, wherein it has been explained as necessary.

We wish to record our gratitude to our colleague Professor Richard P. McNitt, who went through the draft manuscript very carefully. Any residual errors are due to us, and no responsibility resides with him!

Comments on the original manuscript provided by Professor Jürgen Ehlers, an editor of the Lecture Notes in Physics Series, resulted in considerable improvements and are greatly appreciated.

This work was prepared under the aegis of the Research Center for the Engineering of Electronic and Acoustic Materials at the Pennsylvania State University, and we thank the industrial sponsors of the Center for their gracious support.

We also acknowledge the patience of our family members, and dedicate this work to a glorious future for all the children of the world.

*University Park,
Pennsylvania.
March 1989*

*Akhlesh Lakhtakia
Vijay K. Varadan
Vasundara V. Varadan*

TABLE OF CONTENTS

1. Introduction	1
2. Scattering by Helical Ensembles	6
3. Constitutive Equations	13
4. Field Equations	19
5. Reaction, Reciprocity and Duality	22
6. Energy and Momentum	26
7. Bohren's Decomposition	30
8. Reflection and Transmission of Plane Waves	34
9. The Imaging Concept	37
10. Scattering by a Circular Chiral Cylinder	43
11. Scattering by a Chiral Sphere	47
12. Scattering by 3-D Chiral Bodies - The T-Matrix Method	51
13. Infinite-Medium Dyadic Green's Functions for the Electromagnetic Fields	58
14. Vector and Scalar Potentials	66
15. Radiation in Chiral Media	71
16. Equivalence of Sources	80
17. Huygens's Principle and Scattering Formalisms	84
18. Plane Wave Scattering in Chiral Media	90
19. A Scalar Treatment	96
20. Acoustically Chiral Solids	100
21. Selected Dyadic Relations	108
22. Selected Bibliography	110