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318

Bertrand Mercier

An Introduction
to the Numerical Analysis
of Spectral Methods



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EDITORS' PREFACE

This is a translation of report CEA-N-2278, dated 1981, of the French Atomic Energy Commission, titled **Analyse Numérique des Méthodes Spectrales**. The translation was prepared under the auspices of the Institute for Computer Applications in Science and Engineering (ICASE).

We hope that this book will serve as an elementary introduction to the mathematical aspects of spectral methods. The first part of the monograph is a reasonably complete introduction to the theory of Fourier series while the second part lays some foundations for the theory of polynomial expansion methods, in particular Chebyshev expansions.

No monograph of this size can hope to serve as a comprehensive reference to all aspects of spectral methods. The emphasis here is on proving rigorously some fundamental results related to one-dimensional advection and diffusion equations. No applications of the methods are presented and no revisions have been made to account for results subsequent to 1981. The reader interested in recent theory and applications of spectral methods might wish to consult the book by Canuto et al. [5].

May 1988

Nessan Mac Giolla Mhuiris
Mohammed Yousuff Hussaini

AUTHOR'S PREFACE

These notes were written while I was teaching a course on Spectral Methods at the Université Pierre et Marie Curie, Paris, at the request of Professors P.G. CIARLET and P.A. RAVIART, whom I would like to thank here.

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February 1985

B.MERCIER

CONTENTS

INTRODUCTION	1
A. FOURIER SPECTRAL METHOD	
1. Review of Hilbert Bases.....	7
2. Simple Examples of Hilbert Bases.....	9
3. Fourier Series in $L^2(-\pi, \pi)$	14
4. The Uniform Convergence of Fourier Series.....	19
5. The Fourier Series of a Distribution.....	21
6. Periodic Sobolev Spaces.....	26
7. First-Order Equations - The Galerkin Method.....	32
8. Lagrange Interpolation in S_N - The Discrete Fourier Transform.....	43
9. First-Order Equation - The Collocation Method.....	56
10. Time Discretization Schemes.....	62
11. An Advection - Diffusion Equation.....	77
12. The Solution of an Elliptic Problem.....	93
B. POLYNOMIAL SPECTRAL METHODS	
1. A Review of Orthogonal Polynomials.....	97
2. An Introduction to Certain Integration Formulae.....	100
3. The Approximation of a Function by Chebyshev Polynomials.....	106
4. Approximation by the Interpolation Operator.....	122
5. The Solution of the Advection Equation.....	126
6. Time Discretization Schemes.....	137
7. The Use of the Fast Fourier Transform.....	141
8. Solutions of the Heat Equation.....	145
REFERENCES	153