

# Lecture Notes in Physics

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Itamar Pitowsky

Quantum Probability –  
Quantum Logic

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## Preface

This monograph is a result of a course on the conceptual foundations of quantum mechanics, given in 1986 and 1988 to physics graduate students, at the Racach Institute of Physics at the Hebrew University, and in 1987 to philosophy graduate students at the University of Western Ontario.

While preparing for the course, I was struck by the immense number and variety of approaches to the problem of interpretation of quantum mechanics. The variability presents itself not just on the level of ideology – which is understandable – but also on the level of notations and formulations. It seems that people do not quite agree on the proper way to present the questions, or even what the questions themselves are. Being perhaps overly optimistic, I set out to look for a unifying principle; a formulation that will make it possible to present the serious alternative approaches, and compare them, on a fair common ground.

As a first attempt I chose to examine the concept "correlation" and for simplicity to deal with correlations of events, rather than random variables in general. The result was an article, Pitowsky (1986), which is essentially a comparison between the quantum concept of correlation and correlations as conceived in traditional probability theory. I was surprised at the richness and unifying power of the subject, with the result that the article has grown into this monograph.

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The possible range of values of classical correlations is constrained by linear inequalities which can be presented as facets of polytopes, which I call "classical correlation polytopes." These constraints have been the subject of investigation by probability theorists and statisticians at least since the 1930s, though the context of investigation was far removed from physics. The linear constraints in question include Bell's inequalities, Clauser-Horne inequalities and their generalizations. Chapter 2 of this monograph is devoted to the study of classical correlation polytopes, their description in terms of vertices and facets, and their relations to propositional logic and computer science. The subject has applications which transcend quantum physics and even physics in general: It is closely related to the study of the Ising spin model, neural networks, and computational complexity. Some of these applications are indicated in the text, and others, which transcend the scope of this monograph, are indicated in Pitowsky (1988).

Chapter 3 is a similar analysis of quantum correlations in terms of linear inequalities and polytopes. At its center is a theorem which completely characterizes the possible range of values of quantum correlations. The rest of Chapter 3 is devoted to examples where classical constraints are violated by quantum frequencies, most notably the violation of Bell-type inequalities in the Einstein-Podolsky-Rosen experiment.

Classical correlation polytopes are also closely related to classical propositional logic. An argument by analogy, with respect to quantum correlation polytopes, leads directly to quantum logic. Chapter 4 is devoted to the study of quantum logic. In particular I prove that any realistic conception of quantum logic implies a

violation of locality. Chapter 5 is an analysis of the hidden variable approach. It includes a detailed construction of local hidden variable theories, based on an extension of classical probability. This generalizes an earlier work of mine and its extension by S.P. Gudder. In particular I address some objections raised in the physics literature.

On the level of interpretation, the analysis of correlations provides for what I believe to be a fair comparison of four basic approaches: The Copenhagen interpretation, the antirealist view, hidden variable theories and quantum logic. No reference is made to the "many worlds interpretation", and mystical views are mentioned only in passing.

This book is a research monograph, and is not intended as a review or a textbook. Consequently, references are made only to those publications which bear directly on the text. Even so, the scope is quite vast, and I cannot pretend to cover all the relevant material. I thus apologize for any omission which results from my ignorance.

Through the years I have benefited from conversations and correspondence with many colleagues and friends. Many thanks are due to Jeffrey Bub who taught me the basic lessons on the "quantum muddle", to the late Peter Moldauer, to Stanley Gudder, David Mermin, Henry Stapp, Malcolm Forster, Abner Shimony, Arthur Fine, Michael Friedman, Roger Cooke, David Malament, Alan Stairs, David Albert, Yemima Ben-Menachem, Mara Beller, Mark Steiner, Benjamin Weiss, and Mendel Sachs.

Parts of this monograph were written while I was visiting the University of

Western Ontario in the fall of 1986 and 1987. I would like to thank the Department of Philosophy for its hospitality, in particular Ray and Bill Demopolous and Lisa and Michael Daws. I would also like to thank Nancy Weber for preparing this manuscript for publication.

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Last but not least I am grateful to my wife Liora Lurie and my daughters Noga and Michelle for their love, patience and support.

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