

Lecture Notes
in Control and Information Sciences

278

Editors: M. Thoma · M. Morari

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Chunling Du and Lihua Xie

H_∞ Control and Filtering of Two-dimensional Systems

With 32 Figures



Springer

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Cataloging-in-Publication Data applied for
Die Deutsche Bibliothek – CIP-Einheitsaufnahme
Du, Chunling:

H_∞ control and filtering of two-dimensional systems / Chunling Du and Lihua Xie. -
Berlin ; Heidelberg ; New York ; Barcelona ; Hong Kong ; London ; Milan ;
Paris ; Tokyo : Springer, 2002

(Lecture notes in control and information sciences ; 278)

(Engineering online library)

ISBN 3-540-43329-5

ISBN 3-540-43329-5 Springer-Verlag Berlin Heidelberg New York

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<http://www.springer.de>

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Printed in Germany

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Typesetting: Digital data supplied by author. Data-conversion by PTP-Berlin, Stefan Sossna e.K.
Cover-Design: design & production GmbH, Heidelberg
Printed on acid-free paper SPIN 10869692 62/3020Rw - 5 4 3 2 1 0

Preface

Two-dimensional(2-D) signals and systems have vast applications in process control, image and signal processing. The study of 2-D systems has attracted tremendous interest from researchers and practitioners in recent years. Many techniques have been developed for the analysis and synthesis of 2-D systems. This monograph presents research work on H_∞ control and filtering for 2-D discrete systems. We first establish several versions of 2-D Bounded Real Lemma in terms of solution of certain 2-D Riccati difference inequality or equation. Based on the derived Bounded Real Lemma, we consider the H_∞ control and filtering problem of 2-D discrete systems described in state space. Solutions are obtained in terms of algebraic Riccati inequalities (ARIs) or linear matrix inequalities (LMIs). Also, the problem of output feedback stabilization of uncertain 2-D discrete systems is considered and the link between the robust stabilization and H_∞ control for 2-D systems is established. Furthermore, the H_∞ model reduction and deconvolution of 2-D digital systems are approached. The applications of the developed H_∞ control and filtering techniques in signal processing, image processing and process control are demonstrated throughout the monograph.

The work in this monograph was performed in the School of Electrical and Electronic Engineering at Nanyang Technological University, Republic of Singapore and was financially supported by the University Scholarship. We would like to express our appreciation to Professors Yeng Chai Soh and Cishen Zhang for their suggestions and technical collaboration concerning the work presented in this monograph.

Lihua Xie
Chunling Du

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Symbols and Acronyms

2-D:	two-dimensional.
1-D:	one-dimensional.
LMI:	linear matrix inequality.
ARI:	algebraic Riccati inequality.
RDI:	Riccati difference inequality.
RDE:	Riccati difference equation.
\mathcal{Z} :	set of interger numbers.
i, j :	integer-valued vertical and horizontal coordinates.
\mathcal{R}^n :	n -dimensional real Euclidean space.
$\mathcal{R}^{n \times m}$:	set of $n \times m$ real matrices.
I_n :	$n \times n$ identity matrix.
$\text{diag}\{A_1, A_2, \dots, A_n\}$:	block diagonal matrix with A_j (not necessarily square) on the diagonal.
X^T :	transpose of matrix X .
$X^{1/2}$:	a symmetric square root of a matrix $X = X^T \geq 0$, i.e., $X^{1/2} X^{1/2} = X$.
$P \geq 0$:	symmetric positive semidefinite matrix $P \in \mathcal{R}^{n \times n}$.
$P > 0$:	symmetric positive definite matrix $P \in \mathcal{R}^{n \times n}$.

$P \geq Q$:	$P - Q \geq 0$ for symmetric $P, Q \in \mathcal{R}^{n \times n}$.
$P > Q$:	$P - Q > 0$ for symmetric $P, Q \in \mathcal{R}^{n \times n}$.
$\sigma_{\max}(X)$:	maximum singular value of matrix X .
$\text{Ker}(X)$ or $\mathcal{N}(X)$:	null space of linear operator X .
$\mathcal{R}(X)$:	range space of linear operator X .
X^\perp :	matrix satisfying $\mathcal{N}(X^\perp) = \mathcal{R}(X)$ and $X^\perp X^{\perp T} > 0$.
$\ \cdot\ $:	Euclidean vector norm.
$\ w\ _2$:	ℓ_2 -norm of a 2-D discrete-time signal $\{w(i, j)\}$, i.e., $\sqrt{\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \ w(i, j)\ ^2}$.
$\ell_2\{[0, \infty), [0, \infty)\}$:	space of square summable sequences on $\{[0, \infty), [0, \infty)\}$ with values on \mathcal{R}^n .
$z_i (i = 1, 2)$:	2-D z -transform variables.