Lecture Notes in Control and Information Sciences 277

Editors: M. Thoma · M. Morari

Springer
Berlin
Heidelberg
New York
Barcelona
Hong Kong
London
Milan
Paris
Tokyo

A. Sasane

Hankel Norm Approximation for Infinite-Dimensional Systems

With 19 Figures

Springer

Author

Amol Sasane
University of Twente
Signals, Systems and Control Dept.
Faculty of Mathematical Sciences
Postbus 217
7500 AE Enschede
The Netherlands

Cataloging-in-Publication Data applied for
Die Deutsche Bibliothek – CIP-Einheitsaufnahme
Sasane, Amol:
Hankel norm approximation for infinite-dimensional systems / Amol Sasane. -
Berlin ; Heidelberg ; New York ; Barcelona ; Hong Kong ; London ; Milan ;
Paris ; Tokyo : Springer, 2002
(Lecture notes in control and information sciences ; 277)
(Engineering online library)
ISBN 3-540-43327-9

ISBN 3-540-43327-9 Springer-Verlag Berlin Heidelberg New York

Springer-Verlag Berlin Heidelberg New York
a member of BertelsmannSpringer Science + Business Media GmbH

http://www.springer.de

Typesetting: Digital data supplied by author. Data-conversion by PTP-Berlin, Stefan Sossna e.K.
Cover-Design: design & production GmbH, Heidelberg
Printed on acid-free paper SPIN 10869676 62/3020Rw - 5 4 3 2 1 0

Preface

This book is aimed primarily at system theorists working on the approximation of systems described in terms of linear partial differential equations by systems described in terms of linear ordinary differential equations. However, it will be of interest also to functional analysts with an interest in the classical problems of interpolation theory of analytic functions.

Model reduction is an important engineering problem in which one aims to replace an elaborate model by a simpler model without undue loss of accuracy. The accuracy can be mathematically measured in several possible norms and the Hankel norm is one such. The Hankel norm gives a meaningful notion of distance between two linear systems: roughly speaking, it is the induced norm of the operator that maps past inputs to future outputs. It turns out that the engineering problem of model reduction in the Hankel norm is closely related to the mathematical problem of finding solutions to the sub-optimal Nehari-Takagi problem, which we call "the sub-optimal Hankel norm approximation problem" in this book. Although the existence of a solution to the sub-optimal Hankel norm approximation problem has been known since the 1970's, in this book, we give explicit solutions and, in particular, we give new formulae for several large classes of infinite-dimensional systems.

The approach taken in this book is as follows. First we give a "frequency domain" solution to the sub-optimal Hankel norm approximation problem. We start with a complex matrix-valued function G defined on the imaginary axis satisfying certain assumptions. In particular, we demand the existence of a solution to a certain $J-$spectral factorization problem. We then give a solution to the sub-optimal Hankel norm approximation problem in terms of the $J-$spectral factor. Furthermore, we give a parameterization of all solutions in terms of the $J-$spectral factor, the parameterizing set being the unit ball in a certain Hardy space. In this manner we give purely "frequency domain" solutions to the sub-optimal Hankel norm approximation problem in Chapter 4 of this book.

Subsequently, we give "state-space" solutions to the sub-optimal Hankel norm approximation problem for important classes of infinite-dimensional linear systems. In Chapter 5 we consider the case where G is the transfer function of a well-posed linear system given by a triple of operators (A, B, C). Under certain assumptions we solve the sub-optimal Hankel norm approximation problem for $G = C(sI - A)^{-1}B$ by constructing a $J-$spectral factor in terms of the system parameters (A, B, C) and verifying that this constructed $J-$spectral factor satisfies the assumptions demanded in Chapter 4. In this manner, we obtain "state-space" solutions to the sub-optimal Hankel norm approximation problem for two important classes of well-posed linear systems: the smooth Pritchard-Salamon class of exponentially stable infinite-dimensional systems and the class of exponentially stable analytic systems.

Finally, we also solve the sub-optimal Hankel norm approximation problem for certain classes of infinite-dimensional systems with a non-exponentially stable semigroup and for regular linear systems.

This book is a slightly adapted and extended version of my Ph.D. thesis. It is the outcome of a period of four years at the Department of Mathematics of the University of Groningen, The Netherlands. My advisor during this period was Professor Ruth Curtain, and the results in this thesis are the product of cooperation with her. I would like to express my deepest gratitude for this. Finally, I wish to thank my family for everything.

Contents