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M. Inguscio T. W. Hänsch (Eds.)

# The Hydrogen Atom

Precision Physics of Simple Atomic Systems



Springer

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## Foreword by *Norman F. Ramsey*

Studies of atomic hydrogen have been great sources for scientific discovery because of that atom's simplicity. These discoveries began with the Balmer series in 1885 and include atomic structure, early quantum theories of the atom, Dirac relativistic quantum mechanics, the anomalous magnetic moment of the proton (suggesting an internal structure of the proton) and observations of failures of the Dirac theory to correctly predict the hydrogen hyperfine structure, fine structure and anomalous magnetic moment of electron. These failures stimulated the development of the first successful relativistic Quantum Electrodynamics (QED) with renormalization and the first successful gauge field theory.

The delightful and scientifically exciting conference, *Hydrogen Atom 2*, in Italy on the Tuscan coast showed that experimental studies of atomic hydrogen and closely related atoms continue to be sources of new fundamental information, as shown by the reviews and progress reports in this edition.

The absolute frequency of the fundamental  $1S - 2S$  transition in atomic hydrogen has now been measured to 1.8 parts in  $10^{14}$ , an improvement by a factor of  $10^4$  in the past twelve years. This improvement was made possible by a revolutionary new approach to optical frequency metrology with the regularly spaced frequency comb of a mode locked femto-second multiple pulsed laser broadened in a non-linear optical fiber. Optical frequency measurement and coherent mixing experiments have now superseded microwave determination of the  $2S$  Lamb shift and have led to improved values of the fundamental constants, tests of the time variation of the fine structure constant, tests of cosmological variability of the electron-to-proton mass ratio and tests of QED by measurement of  $g-2$  for the electron and muon.

After years of pioneering efforts atomic hydrogen has now been successfully cooled to a sufficiently low temperature for Bose-Einstein Condensation (BEC) and high precision spectroscopy.

With the recent advances in atomic theories and experimental techniques, the value of the information obtained from studies of atoms that are different from but similar to atomic hydrogen have increased. These studies include atomic helium, muonic hydrogen, positronium, muonium, antihydrogen, moderate  $Z$  ions, high  $Z$  ions, antiprotonic atoms and muonic atoms.

# Preface

Despite their intriguing simplicity two-body atomic systems such as the *hydrogen atom* continue to challenge physicists even after more than a century of research. The hydrogen atom has inspired the development of the fundamental theories on which our modern physical understanding of the world is based. Several simple atoms have been thoroughly studied over many decades. The hydrogen atom is the simplest and experimentally best accessible of them. The understanding of its spectra was something of a *Rosetta stone* in unveiling the laws of *Quantum Mechanics* in the first three decades of the twentieth century and – furthermore – was the spark that ignited the development of modern *Quantum Electrodynamics* (QED) after the discovery of the *Lamb shift* and an *anomaly* in the hyperfine structure interval in the ground state half a century ago.

The list of simple atoms accessible now includes a broad range of very different natural and artificial systems: hydrogen, helium, muonium, positronium, various few-electron ions, muonic atoms and exotic atomic systems containing a pion, antiproton etc. While hydrogen atoms form the essential part of our universe, the unstable atoms like muonium do not exist in nature at all. The investigation of simple atoms has provided us with important knowledge on fundamental interactions between the particles these atoms consist of.

Today, the simple atoms are still an important object of study, but nowadays they play a different role. The theory of such atoms, *bound state QED*, is a fruitful training ground for bound state Quantum Chromodynamics (QCD), the theory of *strong interactions*, and for few-body nuclear theory. The study of common atoms, such as hydrogen and deuterium, is opening intriguing new frontiers of higher and higher accuracy through new experimental technology, such as an entirely new approach to optical frequency metrology.

In the cases of muonium, positronium, muonic atoms and multiply-charged ions, the study implies the development of new sources and new detectors. The application of spectroscopic methods is very attractive for pionic and exotic atoms, because of an extremely high (for particle physics) level of accuracy.

The accurate study of some atoms (hydrogen, deuterium, muonium, helium and hydrogen-like carbon) and some free particles (electron, proton, muon) provides us with new highly accurate values of the fundamental physical constants which are important far beyond the physics of simple atoms.

This publication summarizes the progress of the last twenty years and it presents the state of the art in the field. It contains material from two confer-

ences: *Hydrogen Atom* (Pisa, 1988) and *Hydrogen Atom 2: Precision Physics of Simple Atomic Systems*. The latter took place in Castiglione della Pescaia, Italy, from May 31–June 3, 2000. As was the case twelve years ago, it was organized as a satellite meeting to the International Conference on Atomic Physics. *The Hydrogen Atom 2* meeting involved more than one hundred scientists from around the world working on different aspects of the physics of simple atoms, and offered them the opportunity for interdisciplinary exchanges between atomic spectroscopy, atomic theory, nuclear and particle physics, metrology and quantum field theory.

Most of the contributions to the *Hydrogen Atom 2* meeting are presented in this publication. The book consists of twelve review papers devoted to the main topics of the *precision physics of simple atoms*. The CD contains the electronic version of the book and, in addition, the contributed papers and a file with a scanned copy of the conference proceedings of the first *Hydrogen Atom* meeting.

The study of such a simple thing as the hydrogen atom is indeed of general physical interest for a broad audience, while any conference proceedings reporting detailed information in the field may only be of interest to a narrower community. As a result of this, we decided to put the review papers into book form, while the contributed papers based on progress reports and poster presentations have been put onto the compact disk. We gratefully acknowledge Springer-Verlag for their understanding of the special nature of this endeavour and their agreement to promote the *book + CD* edition.

Support from the Max-Planck-Institut für Quantenoptik (MPQ), the European Laboratory for Non-Linear Spectroscopy (LENS), D. I. Mendeleev Institute for Metrology (VNIIM) is gratefully acknowledged by the organizing committee. Our special thanks go to Jürgen Kluge and Klaus Jungmann for their help in organizing the meeting.

The *Hydrogen Atom* meeting of 2000 was the second in the series and we, as the meeting chairmen, would like to gratefully acknowledge efforts by F. Bassani, M. Inguscio and T. W. Hänsch, who initiated the meeting series and gave essential support in the organization of the second *Hydrogen* meeting.

Garching, Germany  
November, 2000

*Savely G. Karshenboim*  
*Francesco S. Pavone*

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