

Foundations of Engineering Mechanics

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Asymptotical Mechanics of Thin-Walled Structures

With 91 Figures and 30 Tables



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Preface

In spite of wide development and application of numerical techniques (especially FEM), in Mechanics of Solids at all and in Mechanics of Thin-walled Structures, in particular, the analytical methods retain fully their significance. Because possibility of exact solution is provided by high symmetry of corresponding equations and boundary problems, it is usually rather an exception, if dealing with real technical problems. On the contrary, asymptotic expansions, being by important class of approximations, imply a weak or strong asymmetry of the system under consideration and are distinguished by genuine universality (as we will see, the former case (weak asymmetry) leads to regular asymptotics, and latter one to singular asymptotics). Asymptotic methods allow to penetrate into essence of the problem and to reveal the possibilities of its decomposition that leads to real understanding. From other side, they provide a rational way of numerical simulation if there is a need in making of calculations by more precise.

It is worth to mention from very beginning that the thin-walled structures manifest, probably, one of the most suitable fields of Mechanics and Physics at all for using the asymptotic approach. This is a consequence of the presence of natural and small enough parameters in corresponding equations. First of all, any model of thin-walled structure is actually certain asymptotics of continuum model of 3D solid and describes adequately the stress-strain state of thin body far enough from boundaries and concentrated forces where 3D effects may be essential. Engineers reinforce usually these regions by special manners for compensation of such effects. Further, even 2D or 1D equations of thin-walled structures contain themselves the additional small parameters reflecting, e.g., a strong asymmetry of stiffness (tension-compression rigidity is much more than bend and twist ones) or geometry of the structure. This leads to appearance of supplemented boundary layers which prolongation depends, e.g. from the ratio of different stiffness. New small parameters arise if dealing with reinforced or composite structures that are usually characterized by strong asymmetry with respect to stiffness in different directions.

So, there are many different aspects that need in asymptotic consideration to understand specificity of thin-walled structures.

Because the main separating like between asymptotic methods is distinction regular or singular perturbation problem, we consider (after introduction of common asymptotic technique in the first Section) main aspects of corresponding procedure with applications to important technical problems (Sections 2 and 3).

Following Sections 4–7 are devoted to systematic consideration of boundary value problems (statical and dynamical, linear and nonlinear) for homogeneous shells. The role of mentioned small parameter (ratio of bend-twist and tension-compression stiffeners, as well as parameters, characterizing strong anisotropy of structurally-orthotropic shells), is fully clarified.

In Sections 8–10 we consider averaging, continualization and homogenization technique for various problems of discrete and continuous media.

Final Sections of handbook are devoted to more special and refined problems that are not usually (or rarely) discussed in Textbooks and even in Monographs. They are: intermediate asymptotics (Section 11), localization (Section 12), Padé approximants (Section 13) and matching the limiting asymptotic approach (Section 14), complex variables in Nonlinear Dynamics (Section 15), and other asymptotic ideas that may be useful in different problems for thin-walled structures (Section 16). Some general questions of asymptotics methods are discussed in Section 17 and Afterwords.

Finally, the book contains numerous results obtained earlier by the authors [21-95, 103-110, 427, 453-460, 531, 532, 637-639, 669, 670].

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