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Operator Functions and Localization of Spectra

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Preface

1. A lot of books and papers are concerned with the spectrum of linear operators but deal mainly with the asymptotic distributions of the eigenvalues. However, in many applications, for example, in numerical mathematics and stability analysis, bounds for eigenvalues are very important, but they are investigated considerably less than asymptotic distributions. The present book is devoted to the spectrum localization of linear operators in a Hilbert space. Our main tool is the estimates for norms of operator-valued functions. One of the first estimates for the norm of a regular matrix-valued function was established by I. M. Gel'fand and G. E. Shilov in connection with their investigations of partial differential equations, but this estimate is not sharp; it is not attained for any matrix. The problem of obtaining a precise estimate for the norm of a matrix-valued function has been repeatedly discussed in the literature. In the late 1970s, I obtained a precise estimate for a regular matrix-valued function. It is attained in the case of normal matrices. Later, this estimate was extended to various classes of nonselfadjoint operators, such as Hilbert-Schmidt operators, quasi-Hermitian operators (i.e., linear operators with completely continuous imaginary components), quasiunitary operators (i.e., operators represented as a sum of a unitary operator and a compact one), etc. Note that singular integral operators and integro-differential ones are examples of quasi-Hermitian operators.

On the other hand, Carleman, in the 1930s, obtained an estimate for the norm of the resolvent of finite dimensional operators and of operators belonging to the Neumann-Schatten ideal. In the early 1980s sharp estimates for norms of the resolvent of nonselfadjoint operators of various types were established, that supplement and extend Carleman's estimates. In this book, we present the mentioned estimates and, as it was pointed out, systematically apply them to spectral problems.

2. The book consists of 19 chapters. In Chapter 1, we present some well-known results for use in the next chapters.

Chapters 2-5 of the book are devoted to finite dimensional operators and functions of such operators.

In Chapter 2 we derive estimates for the norms of operator-valued functions in a Euclidean space. In addition, we prove relations for eigenvalues of finite matrices, which improve Schur's and Brown's inequalities.

Although excellent computer softwares are now available for eigenvalue computation, new results on invertibility and spectrum inclusion regions for finite matrices are still important, since computers are not very useful, in particular, for analysis of matrices dependent on parameters. But such matrices play an essential role in various applications, for example, in the stability and boundedness of coupled systems of partial differential equations. In addition, the bounds for eigenvalues of finite matrices allow us to derive the bounds for spectra of infinite matrices. Because of this, the problem of finding invertibility conditions and spectrum inclusion regions for finite matrices continues to attract the attention of many specialists. Chapter 3 deals with various invertibility conditions. In particular, we improve the classical Levy-Desplanques theorem and other well-known invertibility results for matrices that are close to triangular ones. Chapter 4 is concerned with perturbations of finite matrices and bounds for their eigenvalues. In particular, we derive upper and lower estimates for the spectral radius. Under some restrictions, these estimates improve the Frobenius inequalities. Moreover, we present new conditions for the stability of matrices, which supplement the Rohrbach theorem.

Chapter 5 is devoted to block matrices. In this chapter, we derive the invertibility conditions, which supplement the generalized Hadamard criterion and some other well-known results for block matrices.

Chapters 6-9 form the crux of the book. Chapter 6 contains the estimates for the norms of the resolvents and analytic functions of compact operators in a Hilbert space. In particular, we consider Hilbert-Schmidt operators and operators belonging to the von Neumann-Schatten ideals.

Chapter 7 is concerned with the estimates for the norms of resolvents and analytic functions of non-compact operators in a Hilbert space. In particular, we consider so-called P-triangular operators. Roughly speaking, a P-triangular operator is a sum of a normal operator and a compact quasinilpotent one, having a sufficiently rich set of invariant subspaces. Operators having compact Hermitian components are examples of P-triangular operators.

In Chapters 8 and 9 we derive the bounds for the spectra of quasi-Hermitian operators.

In Chapter 10 we introduce the notion of the multiplicative operator integral. By virtue of the multiplicative operator integral, we derive spectral representations for the resolvents of various linear operators. That representation is a generalization of the classical spectral representation for resolvents of normal operators. In the corresponding cases the multiplicative integral is an operator product.

Chapters 11 and 12 are devoted to perturbations of the operators of the form $A = D + W$, where D is a normal boundedly invertible operator and $D^{-1}W$ is compact. In particular, estimates for the resolvents and bounds for the spectra are established.

Chapters 13 and 14 are concerned with applications of the main results from Chapters 7-12 to integral, integro-differential and differential operators, as well as to infinite matrices. In particular, we suggest new estimates for the spectral radius of integral operators and infinite matrices. Under some restrictions, they improve the classical results.

Chapter 15 deals with operator matrices. The spectrum of operator matrices and related problems have been investigated in many works. Mainly, Gershgorin-type bounds for spectra of operator matrices with bounded operator entries are derived. But Gershgorin-type bounds give good results in the cases when the diagonal operators are dominant. In Chapter 15, under some restrictions, we improve these bounds for operator matrices. Moreover, we consider matrices with unbounded operator entries. The results of Chapter 15 allow us to derive bounds for the spectra of matrix differential operators.

Chapters 16-18 are devoted to Hille-Tamarkin integral operators and matrices, as well as integral operators with bounded kernels.

Chapter 19 is devoted to applications of our abstract results to the theory of finite order entire functions. In that chapter we consider the following problem: if the Taylor coefficients of two entire functions are close, how close are their zeros? In addition, we establish bounds for sums of the absolute values of the zeros in the terms of the coefficients of its Taylor series. They supplement the Hadamard theorem.

3. This is the first book that presents a systematic exposition of bounds for the spectra of various classes of linear operators in a Hilbert space. It is directed not only to specialists in functional analysis and linear algebra, but to anyone interested in various applications who has had at least a first year graduate level course in analysis. The functional analysis is developed as needed.

I was very fortunate to have had fruitful discussions with the late Professors I.S. Iohvidov and M.A. Krasnosel'skii, to whom I am very grateful for their interest in my investigations.

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