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R. Pastor-Satorras M. Rubi A. Diaz-Guilera (Eds.)

# Statistical Mechanics of Complex Networks



Springer

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# Preface

This volume gathers a number of selected contributions from the XVIII Sitges Conference “Statistical Mechanics of Complex Networks”, held on 10–14 June 2002 in Sitges, Barcelona (Spain). The contributions collected provide a general overview of the recent developments in the field of complex networks, concerning in particular their application in physics, biology, and sociology.

The conference was sponsored by institutions that generously provided financial support: DGICYT of the Spanish Government, CIRIT of the Generalitat of Catalonia, Universitat de Barcelona, and Centre Especial de Recerca (CER) Physics of Complex Systems. The city of Sitges allowed us, as usual, to use the Palau Maricel as the lecture hall. We are also very grateful to all those who collaborated in the organization of the event, Drs. C.J. Pérez, A. Pérez-Madrid, M.-C. Miguel, A. Arenas, and Profs. P. Hänggi and A. Vespignani.

Finally, we wish to express our gratitude to all the speakers and participants in the conference, who contributed to create a high scientific level and a very pleasant atmosphere.

Barcelona,  
March 2003

*The Editors*

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