Part II

Experience Rating

In Part I we focused on the overall or average behavior of a homogeneous insurance portfolio, where the claim number process occurred independently of the iid claim size sequence. As a matter of fact, this model disregards the policies, where the claims come from. For example, in a portfolio of car insurance policies the driving skill and experience, the age of the driver, the gender, the profession, etc., are factors which are not of interest. The policyholders generate iid claims which are aggregated in the total claim amount. The goal of collective risk theory is to determine the order of magnitude of the total claim amount in order to judge the risk represented by the claims in the portfolio as time goes by.

Everybody will agree that it is to some extent unfair and perhaps even unwise if every policyholder had to pay the same premium. A driver with poor driving skills would have to pay the same premium as a policyholder who drives carefully and has never caused any accident in his/her life. Therefore it seems reasonable to build an *individual model* for every policyholder which takes his or her claim history into account for determining a premium, as well as the overall behavior of the portfolio. This is the basic idea of *credibility theory*, which was popularized and propagated by Hans Bühlmann in his monograph [19] and in the articles [17, 18]. The monograph [19] was one of the first rigorous treatments of non-life insurance which used modern probability theory. It is one of the classics in the field and has served generations of actuaries as a guide for insurance mathematics.

In Chapter 5 we sketch the theory on *Bayes estimation* of the premium for an individual policy based on the data available in the policy. Instead of the expected total claim amount, which was the crucial quantity for the premium calculation principles in a *portfolio* (see Section 3.1.3), premium calculation in a *policy* is based on the expected claim size/claim number, conditionally on the experience in the policy. This so-called *Bayes estimator* of the individual premium minimizes the mean square deviation from the conditional expectation in the class of all finite variance measurable functions of the data. Despite the elegance of the theory, the generality of the class of approximating functions leads to problems when it comes to determining the Bayes estimator for concrete examples.

For this reason, the class of *linear Bayes* or *credibility estimators* is introduced in Chapter 6. Here the mean square error is minimized over a subclass of all measurable functions of the data having finite variance: the class of *linear* functions of the data. This minimization procedure leads to mathematically tractable expressions. The coefficients of the resulting *linear Bayes estimator* are determined as the solution to a system of linear equations. It turns out that the linear Bayes estimator can be understood as the convex combination of the overall portfolio mean and of the sample mean in the individual policy. Depending on the experience in the policy, more or less weight is given to the individual experience or to the portfolio experience. This means that the data of the policy become *more credible* if a lot of experience about the policy is available. This is the fundamental idea of *credibility theory*. We consider the

basics on linear Bayes estimation in Section 6.1. In Sections 6.2-6.4 we apply the theory to two of the best known models in this context: the Bühlmann and the Bühlmann-Straub models.