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# Springer Tracts in Modern Physics

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Peter Sigmund

# Stopping of Heavy Ions

A Theoretical Approach

With 43 Figures

 Springer

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# Preface

Accelerators for charged particles with a very wide range of masses and charge states have an increasing application range in fundamental physics research, in medical radiology, in materials science and engineering, in micro and nano science and technology, in nuclear fission and fusion technology, and in mass spectrometry. The range of kinetic energies reached at existing accelerators extends from a few electron volts per particle into the  $10^{12}$  electron volt regime. Effective and successful application of such particle beams requires detailed quantitative knowledge of the penetration of charged particles through matter, a field commonly categorized under the discipline of atomic physics.

Following the quest for critical data tabulations, the International Commission on Radiation Units and Measurements (ICRU) has issued tables on stopping powers and ranges for electrons in 1984 (ICRU Report 37) and for protons and alpha particles in 1993 (ICRU Report 49). A followup for ions heavier than helium was commissioned subsequently, and a report committee chaired by the author of this monograph has delivered a draft report which has passed the reviewing procedure of the commission and is going to be published by Oxford University Press in a forthcoming issue of the Journal of the ICRU. That report presents a survey of stopping theory for heavy ions and of experimental techniques and results, a critical discussion of available tabulations and computer codes, and stopping tables for numerous elements and compounds.

Experimental and theoretical activity in heavy-ion stopping has increased during the past decade, stimulated in part by the ICRU report committee. This, in turn, accentuated the need for an extensive review of the field, and especially a summary of the basic theoretical concepts. While a monograph of the breadth of Niels Bohr's legendary review from 1948 has not appeared ever since, the theory part in the forthcoming ICRU report should be able to serve as a useful guide on available knowledge and future research in the field of stopping of heavy ions, if it were to appear here and now. However, production of the full ICRU report has to follow a schedule agreed to between commission and publisher, and this is still going to take some time. For this reason it was found desirable to publish part of the report now. Duplicate publication appears justified since a monograph in Springer Tracts of Modern Physics is likely to attract a rather different readership than an ICRU report.

The structure of this monograph bears evidence of its origin. Considerable care has been devoted to precise definitions of all pertinent physical

quantities, and an attempt has been made to present a balanced view of important developments. Although not a textbook, the monograph should also become useful in a university course on particle penetration. Detailed derivations are not given, but with the exception of elementary common concepts, all quoted results are supported by references to the literature<sup>1</sup>. I tried to select references according to the guiding principle to identify the first or the best research paper or the latest review. I am afraid that this did not prevent references to my own work from being overrepresented. My apologies in advance.

Also the topical coverage is influenced by the primary task issued by the ICRU, with a heavy emphasis on mean energy loss and fluctuation of swift, not too heavy ions in disordered media. Multiple scattering has been included mainly as a phenomenon affecting stopping measurements, while related phenomena such as stopping of channeled ions or of ions interacting with surfaces are mentioned rather briefly.

The author is grateful to the managements of three organizations involved, the ICRU, Springer, and Oxford University Press, for efficiently removing all obstacles toward publication of this material both as a monograph in Springer Tracts and as two chapters in the forthcoming ICRU report. Differences between the two versions are minute, and mainly motivated by the need to make this stand-alone version self-contained. Where appropriate, reference has been made to material contained in the full report.

The author's thanks go to Mitio Inokuti who initiated this project as an ICRU sponsor and took an active interest in its completion, and to several commission members, in particular Paul DeLuca and Stephen Seltzer for a great reviewing effort. Thanks are due to René Bimbot, Hans Geissel and Helmut Paul for extensive feedback to the text, and to Andreas Schinner, with whom I share responsibility for all aspects of binary stopping theory which forms the scientific basis of the tabulations in the forthcoming ICRU report. Valuable advice has been received from Nestor Arista, Martin Berger, Lev Glazov and Allan H. Sørensen, all of whom put considerable effort into reading several drafts. I am particularly grateful to two scientists who are no longer with us: Niels Bohr, whose insight and vision have had an all-dominating influence on the entire field of particle penetration through matter, and to Jens Lindhard, whose guidance, both directly and by example, has been instrumental for my activity in this area.

This work has been supported by the Danish Natural Science Research Council (SNF).

Odense,  
May 2004

*Peter Sigmund*

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<sup>1</sup>The bibliography has been closed on 1 October 2003.

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# List of Symbols and Abbreviations

$a$	adiabatic radius $v/\omega$ .
$a$	screening radius of interatomic potential.
$a_0$	Bohr radius.
$A_1$	mass number of projectile ion.
$A_2$	mass number of target atom.
$a_{sc}$	atomic screening radius.
$B_j$	Bethe parameter $2mv^2/\hbar\omega_j$ .
$c$	speed of light.
$C$	constant 1.1229 in Bohr logarithm.
CKT	convergent kinetic theory.
csda	continuous-slowing-down approximation.
$-dE/\rho d\ell$	mass stopping force.
$-dE/d\ell$	stopping force, stopping power.
$-dE/dx$	stopping force, stopping power.
$\Delta E_{peak}$	most probable energy loss.
$d\mathcal{P}$	probability.
$d\sigma$	differential cross section.
$E$	kinetic energy of projectile.
$E_0$	initial energy of projectile.
$E/A_1$	specific energy.
$F(\alpha, \ell)$	multiple-scattering profile.
$F(\Delta E, \ell)$	energy-loss profile.
$f(\eta)$	function determining differential cross section in Lindhard units.
$f_j$	oscillator strength of frequency $\omega_j$ .
$\hbar$	Planck's constant divided by $2\pi$ .
$I$	mean excitation energy.
$I_1$	$I$ -value entering straggling.
Im	imaginary part.
$J_n$	Bessel function of the first kind.
$k$	absorption coefficient.
$\ell$	path length.
$L$	stopping number.
$L_0$	stopping number omitting shell correction.

XII List of Symbols and Abbreviations

$L_j$	stopping number for $j$ th shell.
LSS	Theory of ion ranges by Lindhard, Scharff and Schiøtt.
$m$	rest mass of electron.
$m$	exponent entering power cross section.
$M_0$	reduced mass.
$M_1$	rest mass of projectile ion.
$M_2$	rest mass of target atom.
$n$	number of target atoms per volume.
$n$	refractive index.
$n_e$	electron density.
$f(\omega), f'(\omega)$	oscillator-strength spectrum.
$P$	projectile momentum.
$p$	exponent in low-velocity stopping force.
$p$	impact parameter.
$P_\ell$	Legendre polynomial.
$P(v, q_1)$	charge fraction.
$q$	exponent entering nuclear scattering cross section.
$q_1$	ion charge number.
$q_{1,\text{eff}}$	effective-charge number.
$\mathbf{Q}, Q_{IJ}$	matrix governing charge fractions.
$R$	range, range along the path.
$\mathbf{R}$	vector range.
$\mathbf{r}$	position vector.
$R$	internuclear distance.
$R$	Rydberg energy.
$\text{Re}$	real part.
$R_\perp$	lateral range.
$r_{\min}$	distance of closest approach.
$R_p$	projected range.
$R_p/R$	projected-range correction, detour factor.
$r_s$	Wigner-Seitz radius.
$S$	stopping cross section.
$S_e$	electronic stopping cross section.
$s(\varepsilon)$	stopping cross section in Lindhard dimensionless units.
$\mathbf{S}, S_{IJ}$	stopping matrix.
$S_n$	nuclear stopping cross section.
$t$	time.
$U$	ionization energy.
$u$	screening function.
$u$	atomic mass unit.
UCA	unitary convolution approximation.
$v$	projectile speed.
$\mathbf{v}$	projectile velocity.
$v_0$	Bohr velocity.

$\mathbf{v}_e$	velocity of target electron.
$W$	straggling parameter.
$w$	energy transfer in single event.
$w_1$	limiting energy loss between multiple- and single-collision regime.
$w_J$	energy transfer to $J$ th loss channel.
$w_{\max}$	maximum energy loss in single event.
$W_n(E)$	Nuclear-straggling parameter.
$x$	penetration depth.
$Z_1$	atomic number of projectile ion.
$Z_2$	atomic number of target atom.
$\alpha$	total deflection angle.
$\alpha$	fine structure constant $1/137$ .
$\langle \dots \rangle$	beam average.
$\beta$	$v/c$ .
$\Delta E$	energy loss at given pathlength.
$\langle \Delta E \rangle$	mean energy loss.
$\Delta E_{\pm 1/2}$	right and left halfwidth of energy-loss spectrum.
$\delta_\ell$	phase shift.
$\Delta L_{\text{Bloch}}$	Bloch correction to stopping number.
$\Delta L_{\text{invBloch}}$	inverse-Bloch correction to stopping number.
$\Delta \mathbf{u}$	velocity change in single collision.
$\Delta \mathbf{v}$	velocity change after penetration.
$\epsilon_0$	vacuum permittivity.
$\epsilon(\mathbf{k}, \omega)$	Lindhard function.
$\epsilon$	Lindhard dimensionless energy unit.
$\eta$	scaled nuclear energy loss.
$\gamma$	Euler's constant = 0.5772.
$\gamma$	$1/\sqrt{1 - v^2/c^2}$ .
$\gamma^2$	effective-charge fraction.
$\gamma$	energy-transfer factor $4M_1M_2/(M_1 + M_2)^2$ .
$\kappa$	Bohr's kappa parameter $2Z_1v_0/v$ .
$\lambda$	coefficient entering nuclear scattering cross section.
$\nu$	species index.
$\phi$	scattering angle in single event.
$\psi$	digamma function, $\psi(x) = d \ln \Gamma(x)/dx$ .
$\rho$	mass density of medium.
$\sigma_J$	cross section for energy loss $w_J$ .
$\sigma^{(1)}$	transport cross section determining stopping force.
$\sigma_B(k, \kappa)$	transport cross section determining energy loss and angular deflection.
$\boldsymbol{\sigma}, \sigma_{IJ}$	cross-section matrix.

XIV List of Symbols and Abbreviations

$\sigma(s)$	transport cross section determining energy-loss spectrum.
$\omega$	resonance frequency.
$\Omega^2$	straggling, energy-loss straggling, variance of energy-loss profile.
$\Omega_B^2$	Bohr straggling.
$\omega_j$	resonance frequency of $j$ th shell.
$\Omega_R^2$	range straggling, variance of range profile.
$\xi$	Bohr parameter $mv^3/Z_1v_0\hbar\omega$ .