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Non-Smooth Dynamical Systems



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The course of true love never did run smooth.

Shakespeare, A midsummer night's dream

Preface

There are many concrete problems in life, and particularly in mechanics and in the engineering sciences, where non-smooth phenomena play an important role: one might think of the noise of a squeaking chalk on a black-board or, sometimes more pleasantly, the sounds of stringed instruments like a violin. More relevant applications include noise generation in railway wheels, the chattering of machine tools, grating brakes, impact print hammers, percussion drilling machines, etc. Physically speaking, these effects often are due to the fact that there are rigid bodies which are in contact (they “stick”), whereas these contact phases are interrupted by “slip” phases during which one of the bodies moves relative to another. In addition to such behaviour mainly induced by friction, there may also be impacts between different parts of the system.

From a mathematical viewpoint, problems of this kind are not easy to handle, since the resulting models are dynamical systems whose right-hand sides are not continuous or not differentiable. In many cases the solutions have to observe additional restrictions that frequently appear in the form of inequality constraints. Since many concepts from classical dynamical systems theory do rely on the smoothness of the underlying system or (semi-) flow, it was necessary to generalize those concepts to cover non-smooth dynamical systems as well, and it turned out that almost always such generalization is a non-trivial issue.

This book is devoted to the analysis of mathematical aspects of non-smooth dynamical systems, and since we are aiming to develop a rigorous theory, we will often have to restrict ourselves to investigating simple model problems for which, however, we will then obtain quite satisfactory results. Besides mathematicians who are interested in dynamical systems or applications, the book also intends to address researchers e.g. from mechanics or the engineering sciences who want to read more on mathematical techniques that are useful for the analysis of non-smooth dynamical systems, and it would be much reward if those people could profit from these notes.

VIII Preface

My Habilitation thesis KUNZE [114] may be considered a first draft of the book, and I'm grateful to T. Küpper for giving me the opportunity to work on this promising and interesting subject of non-smooth dynamical systems. Sincere thanks are due to the priority research program "Dynamik: Analysis, effiziente Simulation und Ergodentheorie" of Deutsche Forschungsgemeinschaft, and to its coordinator B. Fiedler for initiating valuable exchanges between different parts of dynamical systems theory, through many conferences, workshops, etc. For their support and interest in my work I further wish to thank L. Arnold, J. Batt, W.-J. Beyn, M. Brokate, F. Colonius, K. Deimling, J.-P. Eckmann, Ch. Jones, A. Komech, A. Mielke, M. Monteiro Marques, F. Pfeiffer, K. Popp, J.-F. Rodrigues, J. Scheurle, H. Spohn, and E. Zeidler.

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