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(Eds.)

Quantum Magnetism



Springer

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Preface

Putting the quantum into magnetism might, at first sight, seem like stating the obvious; the exchange interactions leading to collective magnetic behavior are, after all, a pure quantum effect. Yet, for many phenomena in magnetism this underlying quantum nature may be safely ignored at least on the qualitative level. The investigation of magnetic systems where quantum effects play a dominant role and have to be accounted for in detail has, over the last decades, evolved to be a field of very active research. On the experimental side, major boosts have come from the discovery of high-temperature superconductivity in the mid-eighties and the increasing ability of solid state chemists to fashion magnetic systems of restricted dimensionality. While high-temperature superconductivity has raised the question of the link between the mechanism of superconductivity in the cuprates and spin fluctuations and magnetic order in one- and two-dimensional spin-1/2 antiferromagnets, the new magnetic materials have exhibited a wealth of new quantum phenomena of interest in their own. In one-dimensional systems, the universal paradigm of Luttinger liquid behavior has come to the center of interest; in all restricted geometries, the interplay of low dimension, competing interactions and strong quantum fluctuations generates, beyond the usual long range ordered states, a wealth of new states of condensed matter, such as valence bond solids, magnetic plateaux, spin liquid states or spin-Peierls states, to name but a few.

The idea for this book arose during a Hereaus seminar on “Quantum Magnetism: Microscopic Techniques For Novel States of Matter” back in 2002, where it was realized that a set of extensive tutorial reviews would address the needs of both postgraduate students and researchers alike and fill a longstanding gap in the literature.

The first three chapters set out to give an account of conceptual problems and insights related to classes of systems, namely one-dimensional (Mikeska and Kolezhuk), two-dimensional (Richter, Schulenburg and Honecker) and molecular (Schnack) magnets.

The following five chapters are intended to introduce to methods used in the field of quantum magnetism, both for independent reading as well as a backup for the first chapters: this includes time-honored spin wave analysis (Ivanov and Sen), exact diagonalization (Lafflorencie and Poilblanc), quantum

field theory (Cabra and Pujol), coupled cluster methods (Farnell and Bishop) and the Bethe ansatz (Klümper).

To close, a more unified point of view is presented in a theoretical chapter on quantum phase transitions (Sachdev) and an experimentally oriented contribution (Lemmens and Millet), putting the wealth of phenomena into the solid state physics context of spins, orbitals and lattice topology.

Aachen, Magdeburg, Liverpool, Manchester
March 2004

Ulrich Schollwöck
Johannes Richter
Damian Farnell
Ray Bishop

Contents

1 One-Dimensional Magnetism

<i>Hans-Jürgen Mikeska, Alexei K. Kolezhuk</i>	1
1.1 Introduction	1
1.2 $S = \frac{1}{2}$ Heisenberg Chain	5
1.3 Spin Chains with $S > 1/2$	22
1.4 $S = \frac{1}{2}$ Heisenberg Ladders	37
1.5 Modified Spin Chains and Ladders	50
1.6 Gapped 1D Systems in High Magnetic Field	59

2 Quantum Magnetism in Two Dimensions: From Semi-classical Néel Order to Magnetic Disorder

<i>Johannes Richter, Jörg Schulenburg, Andreas Honecker</i>	85
2.1 Introduction	85
2.2 Archimedean Lattices	88
2.3 Criteria for Néel Like Order	92
2.4 Magnetic Ground-State Ordering for the Spin Half HAFM on the Archimedean Lattices	100
2.5 Quantum Phase Transitions in 2D HAFM The CaVO $J - J'$ Model and the Shastry-Sutherland Model	125
2.6 Magnetization Process	129

3 Molecular Magnetism

<i>Jürgen Schnack</i>	155
3.1 Introduction	155
3.2 Substances	156
3.3 Experimental Work	159
3.4 Theoretical Techniques and Results	161
3.5 Dynamics	187

4 Spin Wave Analysis of Heisenberg Magnets in Restricted Geometries

<i>Nedko B. Ivanov, Diptiman Sen</i>	195
4.1 Introduction	195
4.2 Dyson-Maleev Formalism	197
4.3 Spin Wave Analysis of Quasi-1D Ferrimagnets	203
4.4 Applications to 2D Heisenberg Antiferromagnets	212

4.5	Modified Spin Wave Theories	219
4.6	Concluding Remarks	223

5 Simulations of Pure and Doped Low-Dimensional Spin-1/2 Gapped Systems

<i>Nicolas Laflorencie, Didier Poilblanc</i>		227
5.1	Introduction	227
5.2	Lanczos Algorithm	228
5.3	Examples of Translationally Invariant Spin Gapped Systems	236
5.4	Lanczos Algorithm for Non-uniform Systems: Application to Doped SP Chains	244
5.5	Conclusion	249

6 Field-Theoretical Methods in Quantum Magnetism

<i>Daniel C. Cabra, Pierre Pujol</i>		253
6.1	Introduction	253
6.2	Path Integral for Spin Systems	255
6.3	Effective Action for Antiferromagnetic Spins Chains	257
6.4	The Hamiltonian Approach	259
6.5	The Non-linear Sigma Model and Haldane's Conjecture	261
6.6	Antiferromagnetic Spin Ladders	264
6.7	Chains with Alternating Bonds	266
6.8	The Two-Dimensional Heisenberg Antiferromagnet	267
6.9	Bosonization of 1D Systems	270

7 The Coupled Cluster Method Applied to Quantum Magnetism

<i>Damian J.J. Farnell, Raymond F. Bishop</i>		307
7.1	Introduction	307
7.2	The CCM Formalism	313
7.3	The XXZ Model	316
7.4	The J - J' Model: A Square-Lattice Model with Competing Nearest-Neighbour Bonds	328
7.5	An Interpolating Kagomé/Triangle Model	334
7.6	The J_1 - J_2 Ferrimagnet	339
7.7	Conclusion	344

8 Integrability of Quantum Chains: Theory and Applications to the Spin-1/2 XXZ Chain

<i>Andreas Klümper</i>		349
8.1	Introduction	349
8.2	Integrable Exchange Hamiltonians	350
8.3	Lattice Path Integral and Quantum Transfer Matrix	353
8.4	Bethe Ansatz Equations for the Spin-1/2 XXZ Chain	359
8.5	Manipulation of the Bethe Ansatz Equations	365
8.6	Numerical Results for Thermodynamical Quantities	370

8.7	Thermal Transport	372
8.8	Summary	377
9 Quantum Phases and Phase Transitions of Mott Insulators		
	<i>Subir Sachdev</i>	381
9.1	Introduction	381
9.2	Coupled Dimer Antiferromagnet	383
9.3	Influence of an Applied Magnetic Field	391
9.4	Square Lattice Antiferromagnet	396
9.5	Triangular Lattice Antiferromagnet	425
9.6	Conclusions	428
10 Spin – Orbit – Topology, a Triptych		
	<i>Peter Lemmens, Patrice Millet</i>	433
10.1	Introduction and General Remarks	433
10.2	Interplay of Structural and Electronic Properties	443
10.3	Copper-Oxygen Coordinations	446
10.4	Vanadium-Oxygen Coordinations	453
10.5	Titanium-Oxygen Coordinations	463
10.6	Conclusion	469
	Index	479

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