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Statistical Mechanics
of Periodic Frustrated
Ising Systems



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To Gudrun Behnke,
for substantial help and encouragement

STATISTICAL MECHANICS OF PERIODIC
FRUSTRATED ISING SYSTEMS

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