

# Lecture Notes in Mathematics

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## One-parameter Semigroups of Positive Operators

Edited by R. Nagel

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## PREFACE

As early as 1948 in the first edition of his fundamental treatise on Semigroups and Functional Analysis, E. Hille expressed the need for "developing an adequate theory of transformation semigroups operating in partially ordered spaces" (l.c., Foreword). In the meantime the theory of one-parameter semigroups of positive linear operators has grown continuously. Motivated by problems in probability theory and partial differential equations W. Feller (1952) and R. S. Phillips (1962) laid the first cornerstones by characterizing the generators of special positive semigroups. In the 60's and 70's the theory of positive operators on ordered Banach spaces was built systematically and is well documented in the monographs of H. H. Schaefer (1974) and A.C. Zaanen (1983). But in this process the original ties with the applications and, in particular, with initial value problems were at times obscured. Only in recent years an adequate and up-to-date theory emerged, largely based on the techniques developed for positive operators and thus recombining the functional analytic theory with the investigation of Cauchy problems having positive solutions to each positive initial value. Even though this development - in particular with respect to applications to concrete evolution equations in transport theory, mathematical biology, and physics - is far from being complete, the present volume is a first attempt to shape the multitude of available results into a coherent theory of one-parameter semigroups of positive linear operators on ordered Banach spaces.

The book is organized as follows.

We concentrate our attention on three subjects of semigroup theory: characterization, spectral theory and asymptotic behavior. By characterization, we understand the problem of describing special properties of a semigroup, such as positivity, through the generator. By spectral theory we mean the investigation of the spectrum of a generator. Asymptotic behavior refers to the orbits of the initial values under a given semigroup and phenomena such as stability.

This program (characterization, spectral theory, asymptotic behavior) is worked out on four different types of underlying spaces:

A. On Banach spaces. Here we present the background for the subsequent discussions related to order.

B. On spaces  $C_0(X)$  ( $X$  locally compact), which constitute an important class of ordered Banach spaces and where our results can be presented in a form which makes them accessible also for the non-expert in order-theory.

C. On Banach lattices, which admit a rich theory and are still sufficiently general as to include many concrete spaces appearing in analysis; e.g.,  $C_0(X)$ ,  $L^p(\mu)$  or  $l^p$ .

D. On non-commutative operator algebras such as  $C^*$ - or  $W^*$ -algebras, which are not lattice ordered but still possess an interesting order structure of great importance in mathematical physics.

In each of these cases we start with a short collection of basic results and notations, so that the contents of the book may be visualized in the form of a  $4 \times 4$  matrix in a way which will allow "row readers" (interested in semigroups on certain types of spaces) and "column readers" (interested in certain aspects) to find a path through the book corresponding to their interest.

We display this matrix, together with the names of the authors contributing to the subjects defined through this scheme:

	I Basic Theory	II Character- ization	III Spectral Theory	IV Asymptotics
A. Banach Spaces	R. Nagel U. Schlotterbeck	W. Arendt H. P. Lotz	G. Greiner R. Nagel	F. Neubrander
B. $C_0(X)$	R. Nagel U. Schlotterbeck	W. Arendt	G. Greiner	A. Grabosch G. Greiner U. Moustakas F. Neubrander
C. Banach Lattices	R. Nagel U. Schlotterbeck	W. Arendt	G. Greiner	A. Grabosch G. Greiner U. Moustakas R. Nagel F. Neubrander
D. Operator Algebras	U. Groh	U. Groh	U. Groh	U. Groh

This "matrix of contents" has been an indispensable guide line in our discussions on the scope and the spirit of the various contributions. However, we would not have succeeded in completing this manuscript, as a collection of independent contributions (personally accounted for by the authors), under less favorable conditions than we have actually met. For one thing, Rainer Nagel was an unfaltering and undisputed spiritus rector from the very beginning of the project. On the other hand we gratefully acknowledge the influence of Helmut H. Schaefer and his pioneering work on order structures in analysis. It was the team spirit produced by this common mathematical background which, with a little help from our friends, made it possible to overcome most difficulties.

We have prepared the manuscript with the aid of a word processor, but we confess that without the assistance of Klaus Kuhn the pitfalls of such a system would have been greater than its benefits.

The authors

## TABLE OF CONTENTS

Part A	<u>One-parameter Semigroups on Banach Spaces</u>	
A-I	<u>Basic Results on Semigroups on Banach Spaces</u> by Rainer Nagel and Ulf Schlotterbeck	1
	1. Standard Definitions and Results .....	1
	2. Standard Examples .....	7
	3. Standard Constructions .....	13
	Notes .....	24
A-II	<u>Characterization of One-parameter Semigroup on Banach Spaces</u>	25
	1. The Abstract Cauchy Problem, Special Semigroups and Perturbation .....	26
	by Wolfgang Arendt	
	2. Contraction Semigroups and Dissipative Operators .....	47
	by Wolfgang Arendt	
	3. Semigroups on $L^\infty$ and $H^\infty$ .....	54
	by H. P. Lotz	
	Notes .....	58
A-III	<u>Spectral Theory of Semigroups on Banach Spaces</u> by Günther Greiner and Rainer Nagel	60
	1. Introduction .....	60
	2. The Fine Structure of the Spectrum .....	63
	3. Spectral Decompositions .....	68
	4. The Spectrum of Induced Semigroups .....	74
	5. The Spectrum of Periodic Semigroups .....	79
	6. Spectral Mapping Theorems .....	82
	7. Weak Spectral Mapping Theorems .....	89
	Notes .....	96
A-IV	<u>Asymptotics of Semigroups on Banach Spaces</u> by Frank Neubrander	98
	1. Stability: Homogeneous Case .....	98
	2. Stability: Inhomogeneous Case .....	112
	Notes .....	115

Part B Positive Semigroups on Spaces  $C_0(X)$

B-I	<u>Basic Results on <math>C_0(X)</math></u>	117
	by Rainer Nagel and Ulf Schlotterbeck	
	1. Algebraic and Order Structure; Ideals and Quotients.....	117
	2. Linear Forms and Duality.....	118
	3. Linear Operators.....	120
B-II	<u>Characterization of Positive Semigroups on <math>C_0(X)</math></u>	122
	by Wolfgang Arendt	
	1. Generators of Positive Semigroups on $C(K)$ .....	123
	2. Lattice Semigroups on $C_0(X)$ .....	135
	3. Semiflows, Flows and Positive Groups .....	143
	Notes .....	162
B-III	<u>Spectral Theory of Positive Semigroups on <math>C_0(X)</math></u>	163
	by Günther Greiner	
	1. The Spectral Bound .....	163
	2. The Boundary Spectrum .....	169
	3. Irreducible Semigroups .....	182
	4. Semigroups of Lattice Homomorphisms .....	192
	Notes .....	202
B-IV	<u>Asymptotics of Positive Semigroups on <math>C_0(X)</math></u>	204
	1. Stability of Positive Semigroups on $C_0(X)$ .....	204
	by Frank Neubrander	
	2. Compact and Quasi-compact Semigroups .....	209
	by Günther Greiner	
	3. A Semigroup Approach to Retarded Differential Equations ..	219
	by Annette Grabosch and Ulrich Moustakas	
	Notes .....	231

Part C Positive Semigroups on Banach Lattices

C-I	<u>Basic Results on Banach Lattices and Positive Operators</u>	233
	by Rainer Nagel and Ulf Schlotterbeck	
	1. Sublattices, Ideals, Bands.....	236
	2. Order Units, Weak Order Units, Quasi-interior Points.....	238
	3. Linear Forms and Duality.....	238
	4. AM- and AL-Spaces.....	239
	5. Special Connections between Norm and Order.....	241
	6. Positive Operators, Lattice Homomorphisms.....	242
	7. Complex Banach Lattices.....	243
	8. The Signum Operator .....	245
	9. The Center of $L(E)$ .....	246
C-II	<u>Characterization of Positive Semigroups on Banach Lattices</u>	247
	by Wolfgang Arendt	
	1. Positive Contraction Semigroups and Bounded Generators ...	248
	2. Kato's Inequality .....	256
	3. A Characterization of Generators of Positive Semigroups ..	260
	4. Domination of Semigroups .....	269
	5. Semigroups of Disjointness Preserving Operators .....	281
	Notes .....	290
C-III	<u>Spectral Theory of Positive Semigroups on Banach Lattices</u>	292
	by Günther Greiner	
	1. The Spectral Bound .....	292
	2. The Boundary Spectrum .....	296
	3. Irreducible Semigroups .....	306
	4. Semigroups of Lattice Homomorphisms .....	320
	Notes .....	331
C-IV	<u>Asymptotics of Positive Semigroups on Banach Lattices</u>	333
	1. Stability of Positive Semigroups on Banach Lattices .....	334
	by Günther Greiner and Frank Neubrander	
	2. Convergence of Positive Semigroups .....	342
	by Günther Greiner and Rainer Nagel	
	3. A Semigroup Approach to Retarded Equations .....	356
	by Annette Grabosch and Ulrich Moustakas	
	Notes .....	367



Part D Positive Semigroups on  $C^*$ - and  $W^*$ -Algebras

by Ulrich Groh

D-I	<u>Basic Results on Semigroups and Operator Algebras</u>	369
	1. Notations .....	369
	2. A Fundamental Inequality for the Resolvent .....	370
	3. Induction and Reduction .....	374
D-II	<u>Characterization of Positive Semigroups on <math>W^*</math>-Algebras</u>	376
	1. Positive Semigroups on Properly Infinite $W^*$ -Algebras .....	376
D-III	<u>Spectral Theory of Positive Semigroups on <math>W^*</math>-Algebras and their Preduals</u>	379
	1. Spectral Theory for Positive Semigroups on Preduals .....	379
	2. Spectral Properties of Uniformly Ergodic Semigroups .....	391
	Notes .....	398
D-IV	<u>Asymptotics of Positive Semigroups on <math>C^*</math>- and <math>W^*</math>-Algebras</u>	400
	1. Stability of Positive Semigroups .....	400
	2. Stability of Implemented Semigroups .....	403
	3. Convergence of Positive Semigroups .....	406
	4. Uniform Ergodic Theorems .....	419
	Notes .....	425
	Bibliography .....	427
	Table of Symbols .....	453
	Subject Index .....	456