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Stability Problems for Stochastic Models

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Foreword

The 6 th seminar on Problems of Stochastic models stability took place in Moscow from 19 to 27 April 1982. The previous seminars were organized as follows: in Leningrad and Vilnius (1974), in Moscow (1977), in Palanga, Lithuanian SSR (1979), and in Panevežis, Lithuanian SSR (1980). The material of these seminars was published in four special proceedings [1] - [4] . Short information about the 4th and 5th seminars and summaries of some of the papers were published in [5] and [6] .

The 6 th seminar was organized by the Steklov Mathematical Institute of the Academy of Sciences of USSR (MIAN), the International Research Institute of Control Problems (MNIIPU), which assumed the greatest part of the care of organizing the seminar, and the Institute for Systems studies (VNIISI).

The organizing committee consisted of Academician Prof.S.V.Emeljanov - Chairman (MNIIPU), Prof. V.M.Zolotarev - Vice-chairman (MIAN), Prof. V.V.Kalashnikov (VNIISI) and Dr. V.I.Lutkov - Secretary (MNIIPU). There were 47 reports submitted by representatives of well known mathematical centres of seven countries. Those reports not included in this volume are listed by title.

The present proceedings include papers based on the reports submitted at the seminar, as well as a paper by the Japanese mathematician R. Shimizu (presented at the Soviet-Japanese symposium on Probability theory, held at the end of August 1982 in Tbilisi), and a paper by the Greek mathematician J. Panaretos (delivered at the International Conference on Probability theory, held in Vilnius in June 1981).

A characteristic feature of all six seminars on Problems of Stochastic models stability was the great variety of topics in the papers delivered.

Though most of the reports have always been devoted to the main topic of the seminar, the papers presented here featured such problems as characterization of distributions, investigations of probability metrics, characteristic functions, etc. The point is that the respective interests within probability theory and its application of most of the participants are often quite far apart from each other. The fact that the seminar managed, during its existence, not only to retain, but also increase the number of "permanent" participants can be explained first and foremost by a tradition, created at the seminar, according to which every participant could pose any problem in which he had taken a lively interest. The discussion of the contributions to our seminars was performed from the viewpoint of their general tendency, i.e. from one of the general conceptions of stability. Of course the participants interest in the stability problems isn't of itself sufficient for such a practice to be a success. It is necessary to have also some general conceptual and methodological approach to the stability phenomenon. During the existence of the seminar such a general approach was outlined and here we give a brief review of it.

The notion of stochastic model stability can be explained within the framework of the following scheme. Let $\mathscr{X}=\{\bar{X}\}$ and $\mathscr{Y}=\{\bar{Y}\}$ be sets of random variables defined on a probability space and taking values from the corresponding measurable spaces $(U,\mathfrak{B}_U)$ and $(V,\mathfrak{B}_V)$. Suppose that some mapping $\mathbb{F}$ of the set $\mathscr{X}$ into $\mathscr{Y}$ is given and the sets $\mathscr{A}\subseteq\mathscr{X}$ and $\mathscr{B}\subseteq\mathscr{Y}$ describe the conditions imposed on the random variables mapping from $\mathscr{X}$ and their images in $\mathscr{Y}$ (see Fig.1).

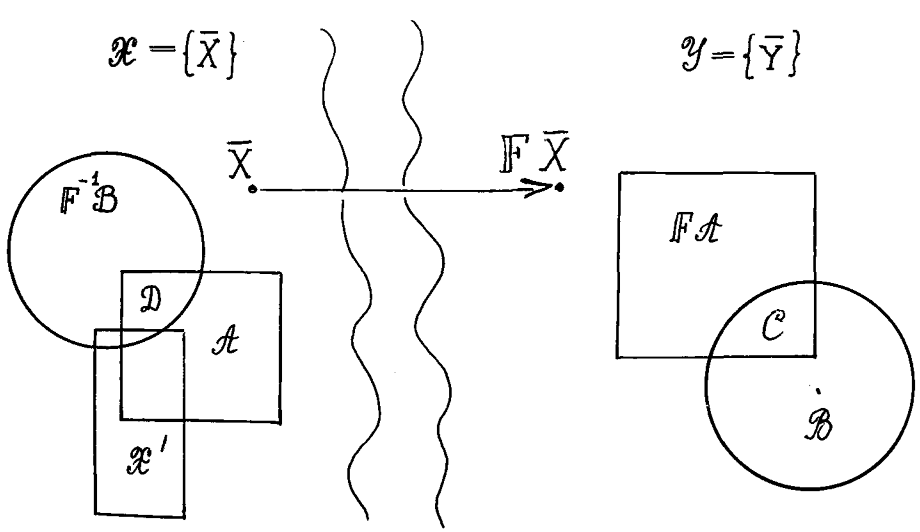

Fig. 1

The problems connected with the triplet $(\mathscr{A},\mathscr{B},\mathbb{F})$ are divided into direct problems in which one is interested in the properties of the values from the set $\mathscr{C}=\mathscr{B}\cap\mathbb{F}\mathscr{A}$, and the inverse ones in which the object of the investigation is the set $\mathscr{D}=\mathscr{A}\cap\mathbb{F}^{-1}\mathscr{B}$ (here $\mathbb{F}^{-1}\mathscr{B}$ denotes the pre-image of the set $\mathscr{B}$).

Note that two important particular cases of this model correspond to large classes of problems when $\mathcal{A} = \mathcal{X}$ or $\mathcal{B} = \mathcal{Y}$.

The concept of stability of direct and inverse problems is related to certain probability metrics μ and ν defined on the spaces $\mathcal{X}$ and $\mathcal{Y}$ and to a certain set $\mathcal{X}' \subseteq \mathcal{X}$.

Under the notion of the probability metric in $\mathcal{X}$ we mean any functional $0 \leq \mu(\bar{X}, \bar{X}') \leq \infty$ defined on the set of joint distributions of all pairs of the random variables $\bar{X}, \bar{X}'$ from $\mathcal{X}$ and having the pseudometrics properties in $\mathcal{X}$, i.e. the symmetry property, the triangle inequality and the property $P(\bar{X} = \bar{X}') = 1 \Longrightarrow \mu(\bar{X}, \bar{X}') = 0$. All distances used in probability theory are, obviously, probability metrics, however distances like $|E(X-X')|$ also belong to them (if U is a Banach space).

If we consider

$$\mu(\bar{X}', \mathcal{A}) = \inf\{\mu(\bar{X}', \bar{X}) : \bar{X} \in \mathcal{A}\}$$

as the usual distance between an element and a set we can say that the direct problem, connected with the triplet $(\mathcal{A}, \mathcal{B}, \mathbb{F})$ is stable with respect to the metrics μ, ν and the set $\mathcal{X}'$ if the relation

$$\varepsilon = \mu(\bar{X}', \mathcal{A}) + \nu(\mathbb{F}\bar{X}', \mathcal{B}) \longrightarrow 0$$

implies

$$\delta = \nu(\mathbb{F}\bar{X}', \mathcal{C}) \longrightarrow 0,$$

where $\mathcal{C} = \mathcal{B} \cap \mathbb{F}\mathcal{A}$. The implication $\varepsilon \to 0 \Longrightarrow \delta \longrightarrow 0$ corresponds to the notion of a qualitative stability of the model, while any estimate $\delta \leq \Psi(\varepsilon)$, where $\Psi(\varepsilon) \to 0$, when $\varepsilon \to 0$ is a solution of a quantitative stability problem of the same model.

Such a comprehension of the mathematical model stability is very capacious and is able to include practically any problem of approximating type. For instance, it includes the general notion of robustness, proposed by Humpel [7] and the notion of differential equation stability (according to Liapunov) with random perturbations, developed in the works of Kushner [8] , etc.

It turns out that in contrast to the quantitative stability, the qualitative stability depends very little on the structure of the model. It can be treated as a consequence of such general properties like the closeness of the sets $\mathcal{A}$ and $\mathcal{B}$, the continuity of the mapping $\mathbb{F}$ and the relative compactness of the set $\mathcal{X}'$ with respect to the implemented metrics (see [9]). This important fact is actually the background on which the specialists in different branches of the probability theory could unite their efforts in investigating the stochastic model stability.

At the same time, the main interest (from the view point of the applications) is in the quantitative estimates of the stability. For instance, they can show us the boundaries of applicability of the model in hand (see [10]). Obtaining quantitative estimates of stability, especially of the correct order, is usually the far more complicated problem than the one of qualitative analysis of stability. It requires more details about the structural peculiarities of the model.

The metric treatment of stability problem assumes a good knowledge of different properties of probability metrics. Though the theory of probability metrics is still at the stage of development and accumulation of facts, one can already speak about the existence of a common methodological approach to the solution of the stability problems. We call it the metric distances method. In stochastics, probability metrics have been used for a long time. However, the choice of the metrics to be used in approximation problems is determined by topological reasons, computational conveniencies or just by the established tradition.

The method of metric distances is the tool for solving the quantitative stability problems, i. e. the problems of obtaining estimates of the form $\delta \leq \varphi(\varepsilon)$ on the set $\mathfrak{X}'$. The main idea of the method consists in finding such a pair of metrics μ^*, ν^* for which the analogical inequality $\delta^* \leq \varphi_1(\varepsilon^*)$ (on $\mathfrak{X}'$) can be obtained more easily. After we have to look for the estimates of the type: $\mu \leq \varphi(\mu^*)$, $\nu^* \leq \varphi_1(\nu)$, where φ, φ_1 are nondecreasing continuous functions,such that $\varphi(0)=0$, $\varphi_1(0)=0$. Note that the functions ψ, ψ_1 possess the same properties. In order to simplify the explanation we shall consider the case of a fixed metric $\nu^* = \nu$ and possibly varying metric μ .

Let the metrics traditionally used be the metrics $\mu_1, \mu_2, \dots$ Consider a collection of approximation problems, i.e. problems of stability of the corresponding models.

The metrics μ_1, μ_2 and eight problems in the collection considered could be schematically represented by circles (see Fig.2). Besides, the solutions of these problems, in terms of the metrics mentioned, can be represented by lines connecting the circles = problems with the corresponding circles = metrics.

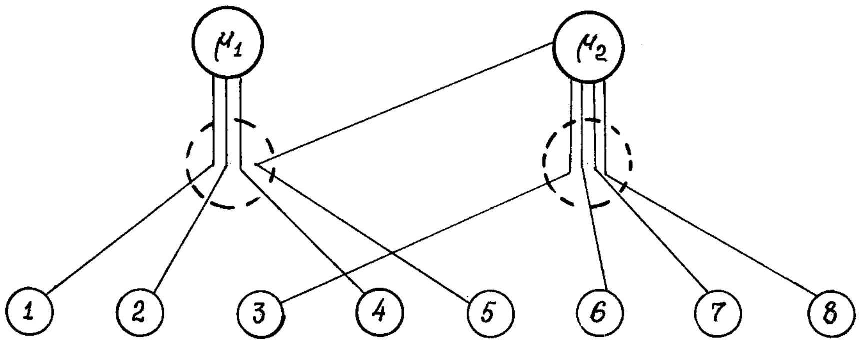

Fig.2

The first principle of the metrics distance method is the following. For every problem of stability we have to choose such metrics in $\mathcal{X}$ and $\mathcal{Y}$ which are the most convenient for its solution, i.e. the best fitted to its structural peculiarity.

Suppose that for the problems numbered 1, 2, 4, 5 such a "natural" metric turned out to be the metric μ_1^* while for the problems 3,6,7,8 a "natural" one proved to be the metrics μ_2^*.

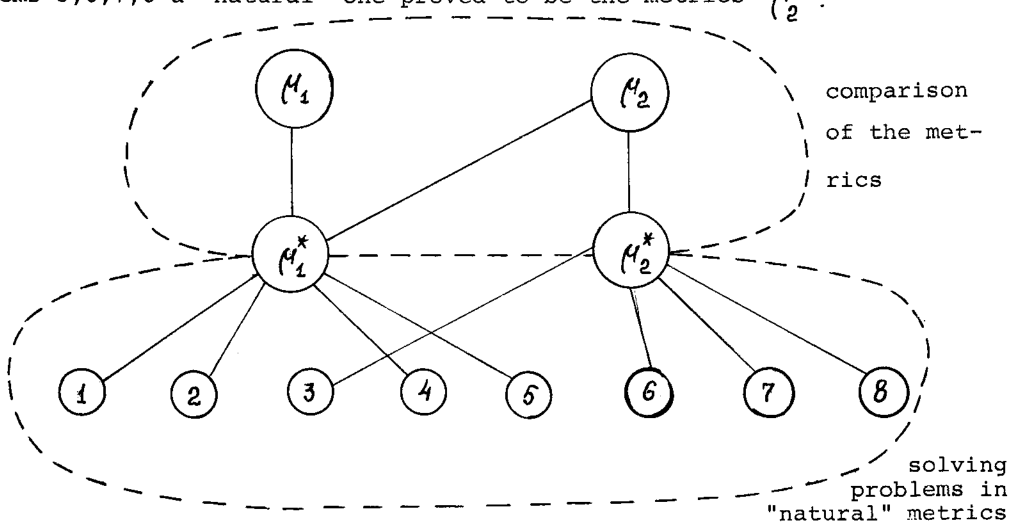

Fig. 3

The basic feature of "natural" metrics is that with their help the solution of the stability problem becomes essentially simpler.

The next step after solving the stability problems with the help of their "natural" metrics is the comparison of the metrics μ_1^*, μ_2^* with the metrics μ_1, μ_2, i.e. the mutual estimation of the first and the second groups of metrics. This step actually forms the second principle of the metric distances method.

Of course, the metric distances method is not able to remove the principal dificulties one is faced when solving the stability problems. It only transfers them by concentrating them in a separate problem of metrics comparison, which is not connected with the initial stability problems. The advantage of such an approach is evident (see Fig.3).

We would like to illustrate the main idea of the metric distances method by the following example.

Let $X_1, X_2, \ldots, X_n$ be independent random variables with the common distribution function $F(x)$ and Y be a random variable with the distribution function

$$G(x) = exp\,(-x^{-\alpha}),\quad x > 0,\ \alpha > 0.$$

Let us form the random variable

$$Z_n = n^{-1/\alpha}\,(X_1 \circ X_2 \circ \ldots \circ X_n),$$

where $x \circ y = max\,(x, y)$ is a binary commutative and associative operation in R^1. Denote by $H(x)$ the distribution function of the random variable Z_n.

It is well known (B.V.Gnedenko [11]) that, under some conditions about the behavior of the "positive tails" of the distribution function $F(x)$, the distribution function $G(x)$ is the limit function of the distributions $H(x)$ when n is infinitely increasing. It is not difficult to check that the random variable Y has the following property. For each $n \geq 1$

(1) $$Y \overset{d}{=} n^{-1/\alpha}\,(Y_1 \circ Y_2 \circ \ldots \circ Y_n),$$

where $Y_1, Y_2, \ldots, Y_n$ are independent random variables identically distributed with Y, and the symbol $\overset{d}{=}$ denotes the equality of distributions.

Consider the problem of approximating the function $H(x)$ by the function $G(x)$ as a direct problem of the stability of the property (1). For that purpose, let us choose as $\mathfrak{X}$ and $\mathfrak{Y}$ the sets of random variables taking values from $U = R^n$ and $V = R^1$ respectively. For each vector

$\bar{x} = (x_1, x_2, \ldots, x_n) \in R^n$ we shall determine the mapping by the equality

$$\mathbb{F}\bar{x} = n^{-1/\alpha}(x_1 \circ x_2 \circ \ldots \circ x_n).$$

Let us choose the set $\mathcal{B} = \mathcal{Y}$ and the set $\mathcal{A} \subset \mathcal{X}$ as the set of all vectors $X = (X_1, X_2, \ldots, X_n)$ with independent and identically distributed components satisfying the condition $\mathbb{F}\bar{X} \overset{d}{=} X_1$ where the random variable X_n has the distribution $G(x)$.

We shall choose in the spaces $\mathcal{X}$ and $\mathcal{Y}$ the following metrics μ and ν

$$\mu(\bar{X}', \bar{X}'') = max\{\nu(X_j', X_j'') : j = 1, 2, \ldots, n\},$$

$$\nu(Y', Y'') = sup\{|x|^s |F_{Y'}(x) - F_{Y''}(x)| : x \in R^1\},$$

where $F_X(x)$ denotes the distribution function of the random variable X and $s > \alpha$ is some fixed number. Finally, let us choose as $\mathcal{X}'$ the set of vectors $\bar{X}' = (X_1', X_2', \ldots, X_n')$ with independent components having a common distribution function $F(x)$ which satisfies the conditions $F(+0) < 1$ and

(2) $$|F(x) - G(x)| = O(|x|^{-s}) \quad \text{for } |x| \to \infty.$$

Because of $\mathcal{B} = \mathcal{Y}$ we have $\mathcal{C} = \mathbb{F}\mathcal{A}$ and $\nu(\mathbb{F}\bar{X}, \mathcal{B}) = 0$ for any $\bar{X} \in \mathcal{X}$. That is why, taking into account the properties of the vectors from the sets $\mathcal{A}$ and $\mathcal{X}'$ we can state that for any $\bar{X}' \in \mathcal{X}'$

$$\varepsilon = \mu(\bar{X}', \mathcal{A}) = \nu(X_1', Y) < \infty$$

where X_1' and Y are random variables with distribution functions $F(x)$ and $G(x)$ respectively, and

$$\delta = \nu(\mathbb{F}\bar{X}', \mathcal{C}) = \nu(\mathbb{F}\bar{X}', \mathbb{F}\mathcal{A}) = \nu(\mathbb{F}\bar{X}', Y).$$

Since the vector $\bar{X} = (X_1, \ldots, X_n)$ which consists of the initial random variables, has the same distribution as the vector $\bar{X}' \in \mathcal{X}$ we have

$$\varepsilon = \nu(X_1, Y), \quad \delta = \nu(Z_n, Y).$$

The problem of estimating the exactness of the approximation of the function $H(x)$ by $G(x)$ turns out to be equivalent to the problem of quantitative estimation of the stability of property (1), i.e. a construction of the estimation δ with the help of $\mathcal{E}$.

The chosen metric ν belongs to the so called "ideal metrics" (see Zolotarev [4]) and has the following properties:

Let X' and X'' be such random variables that the distance $\nu(X', X'')$ is finite and W is a random variable which does not depend on them. Then

1°. $\nu(X' \circ W, X'' \circ W) \leq \nu(X', X'')$,

2°. For each constant $c > 0$

(3) $$\nu(cX', cX'') = c^s \nu(X', X'').$$

As an elementary corollary of property 1° and the triangle inequality we get the inequality

(4) $$\begin{aligned} &\nu(X_1 \circ \ldots \circ X_n, Y_1 \circ \ldots \circ Y_n) \leq \\ &\leq \nu(X_1, Y_1) + \ldots + \nu(X_n, Y_n) = n\nu(X_1, Y). \end{aligned}$$

Thus, from (3) and (4) we obtain

$$\begin{aligned} \delta &= \nu(n^{-1/\alpha}(X_1 \circ \ldots \circ X_n), n^{-1/\alpha}(Y_1 \circ \ldots \circ Y_n)) = \\ &= n^{-s/\alpha} \nu(X_1 \circ \ldots \circ X_n, Y_1 \circ \ldots \circ Y_n) \leq n^{1-s/\alpha} \varepsilon. \end{aligned}$$

This way, since the distribution $F(x)$ of the independent random variables $X_1, \ldots, X_n$ satisfies condition (2) we get an estimate of the exactness of the approximation of the distribution $H(x)$ by the distribution $G(x)$ in the metric ν :

(5) $$\nu(Z_n, Y) \leq n^{1-s/\alpha} \nu(X_1, Y) = \varepsilon n^{1-s/\alpha}.$$

If we want to obtain an analogous estimate in terms of the Lévy metric L or in terms of the uniform metric ρ , then we have to solve the problem of comparing these metrics with the metric ν Let us do it.

The Levy distance between the distribution functions $H(x)$ and $G(x)$ is defined as the length of the side of the maximal square which can be imbedded between the complimented graphs of the functions F and G. Whence

$$\nu(Z_n,Y)=\sup\{|x|^s|H(x)-G(x)|: x\in R^1\} \geq L^{1+s}(Z_n,Y)\cdot 2^{-s}.$$

Thus

$$L(Z_n,Y)\leq (2^s\varepsilon)^{1/(1+s)} n^{-\frac{s-\alpha}{\alpha(1+s)}} \tag{6}$$

The distribution $G(x)$ is absolutely continuous with

$$K_\alpha = \sup\{G'(x): x\in R^1\} =$$

$$= \alpha \exp\left\{\left(1+\frac{1}{\alpha}\right)\left(\log\left(1+\frac{1}{\alpha}\right)-1\right)\right\}.$$

In this case the distance ϱ can be estimated by metric L as follows

$$\varrho(Z_n,Y)\leq (1+K_\alpha)L(Z_n,Y).$$

So we obtain the desired estimate in terms of the metric ϱ:

$$\varrho(Z_n,Y)\leq (1+K_\alpha)(2^s\varepsilon)^{\frac{1}{1+s}} n^{-\frac{s-\alpha}{\alpha(1+s)}}. \tag{7}$$

With the help of examples it can be shown that the order of the estimations (5)-(7), in relation with n, is precise.

Let us note that in the generalized statement of the problem involved when the number of the random variables X_j is a random variable N not depending on X_j, it can be solved also very easily. It is enough to note that for every random variable $N\geq 1$ not depending on Y_j the equality (1) is preserved, i.e.

$$Y \stackrel{d}{=} N^{-1/\alpha}(Y_1\circ\ldots\circ Y_N). \tag{8}$$

Using this property and the estimate (5) we can write the following inequality for the distance ν between the distribution of the random variable

$$Z_N = N^{-1/\alpha}(X_1 \circ \dots \circ X_N)$$

and the distribution of the random variable Y :

$$\nu(Z_N, Y) \leq \sum_{k=1}^{\infty} P(N=k)\, \nu(Z_k, Y) \leq$$

$$\leq \varepsilon \sum_{k=1}^{\infty} P(N=k)\, k^{1-s/\alpha} = \varepsilon E N^{1-s/\alpha}$$

So we get estimates analogical to the estimates (6),(7) where $EN^{1-s/\alpha} \to 0$ if $N \xrightarrow{P} \infty$.

I am convinced that the method of metric distances has great potential capabilities in different fields of probability and statistics and can become a real rival of the traditional methods.

I hope that the material of the present proceedings will help to draw the specialists' attention to the problems of stochastic models stability.

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