

# Lecture Notes in Physics

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Kurt Sundermeyer

## Constrained Dynamics

with Applications to  
Yang-Mills Theory, General Relativity,  
Classical Spin, Dual String Model

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