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Weighted Energy Methods in Fluid Dynamics and Elasticity

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PREFACE

As is known, by energy methods (EM) one denotes a suitable mathematical device for deriving estimates of solutions to differential equations. The name of the method is due to the fact that it is usually founded upon "conservation laws" which must be obeyed by solutions. A typical example of EM, and maybe one of the earliest, is given by the approach introduced at the end of the nineteenth century by the Russian mathematician A.M. Liapounov for studying the stability of solutions to ordinary differential equations. Actually, this approach is based on the existence of a suitable functional which must be positive and non-increasing in time (the so-called "Liapounov functional"). Another no less important example of EM is furnished by the method proposed by J. Serrin in 1959 and successively generalized and deepened by D.D. Joseph and his co-workers, for studying nonlinear stability of viscous incompressible flows in bounded domains. Roughly speaking, the method consists in forming the "kinetic energy" of perturbations to a given basic flow and in studying its behavior in time (see Chapter I). Of course, there are several other examples of applicability of EM, such as uniqueness, existence, etc., and for some of them the reader is referred to the papers cited throughout these Notes.

Aside from the above EM, there are the so-called weighted energy methods (WEM). Unlike the former, the latter explicitly involve the use of suitable auxiliary functions (weight functions) whose task may vary from case to case, depending on the kind of problem one is dealing with. In fluid dynamics, they were originally introduced by the writers in 1975 for studying uniqueness of unsteady viscous flows in unbounded regions. By using suitable spatially weighted norms, they were able to prove uniqueness without prescribing the velocity field at large spatial distances. Since then, WEM have been widely applied to other branches of mathematical physics (magnetohydrodynamics, elasticity, viscoelasticity, etc.) and used to solve different kinds of questions such as existence, continuous data dependence and stability as well. Moreover, they turn out to be useful also in investigating well-posedness problems in bounded domains.

The aim of these Notes is to give an account of WEM and to show how they work in solving several problems arising in fluid dynamics (Part One) and elasticity (Part Two). Of course, we did not consider all possible applications of the method and, among others, we left out important questions such as uniqueness and continuous data dependence of viscuous incompressible flows in unbounded domains; however, we refer the interested reader to the recent Monograph of B. Straughan (Ref. 5 to Chapter II).

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