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Krzysztof C. Kiwiel

Methods of Descent for Nondifferentiable Optimization

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Author

Krzysztof C. Kiwiel
Systems Research Institute, Polish Academy of Sciences
ul. Newelska 6, 01-447 Warsaw, Poland

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PREFACE

This book is about numerical methods for problems of finding the largest or smallest values which can be attained by functions of several real variables subject to several inequality constraints. If such problems involve continuously differentiable functions, they can be solved by a variety of methods well documented in the literature. We are concerned with more general problems in which the functions are locally Lipschitz continuous,but not necessarily differentiable or convex. More succintly, this book is about numerical methods for nondifferentiable optimization.

Nondifferentiable optimization, also called nonsmooth optimization, has many actual and potential applications in industry and science. For this reason, a great deal of effort has been devoted to it during the last decade. Most research has gone into the theory of nonsmooth optimization, while surprisingly few algorithms have been proposed, these mainly by C.Lemaréchal, R.Mifflin and P.Wolfe. Frequently such algorithms are conceptual, since their storage and work per iteration grow infinitely in the course of calculations. Also their convergence properties are usually weaker than those of classical methods for smooth optimization problems.

This book gives a complete state-of-the-art in general-purpose methods of descent for nonsmooth minimization. The methods use piecewise linear approximations to the problem functions constructed from several subgradients evaluated at certain trial points. At each iteration, a search direction is found by solving a quadratic programming subproblem and then a line search produces both the next improved approximation to a solution and a new trial point so as to detect gradient discontinuities. The algorithms converge to points satisfying necessary optimality conditions. Also they are widely applicable, since they require only a weak semismoothness hypothesis on the problem functions which is likely to hold in most applications.

A unifying theme of this book is the use of subgradient selection and aggregation techniques in the construction of methods for nondifferentiable optimization. It is shown that these techniques give rise in a totally systematic manner to new implementable and globally convergent modifications and extensions of all the most promising algorithms which have been recently proposed. In effect, this book should give the reader a feeling for the way in which the subject has developed and is developing, even though it mainly reflects the author's research.

This book does not discuss methods without a monotonic descent (or ascent) property, which have been developed in the Soviet Union.

The reason is that the subject of their effective implementations is still a mystery. Moreover, these subgradient methods are well described in the monograph of Shor (1979). We refer the reader to Shor's excellent book (its English translation was published by Springer-Verlag in 1985) for an extensive discussion of specific nondifferentiable optimization problems that arise in applications. Due to space limitations, such applications will not be treated in this book.

In order to make the contents of this book accessible to as wide a range of readers as possible, our analysis of algorithms will use only a few results from nonsmooth optimization theory. These, as well as certain other results that may help the reader in applications, are briefly reviewed in the introductory chapter, which also contains a review of representative existing algorithms. The reader who has basic familiarity with nonsmooth functions may skip this chapter and start reading from Chapter 2, where methods for unconstrained convex minimization are described in detail. The basic constructions of Chapter 2 are extended to the unconstrained nonconvex case in two fundamentally different ways in Chapters 3 and 4, giving rise to competitive methods. Algorithms for constrained convex problems are treated in Chapter 5, and their extensions to the nonconvex case are described in Chapter 6. Chapter 7 presents new versions of the bundle method of Lemaréchal and its extensions to constrained and nonconvex problems. Chapter 8 contains a few numerical results.

The book should enable research workers in various branches of science and engineering to use methods for nondifferentiable optimization more efficiently. Although no computer codes are given in the text, the methods are described unambiguously, so computer programs may readily be written.

The author would like to thank Claude Lemaréchal and Dr. A.Ruszczyński for introducing him to the field of nonsmooth optimization, and Prof. K.Malanowski for suggesting the idea of the book. Without A.Ruszczyński's continuing help and encouragement this book would not have been written. Part of the results of this book were obtained when the author worked for his doctoral dissertation under the supervision of Prof. A.P.Wierzbicki at the Institute of Automatic Control of the Technical University of Warsaw. The help of Prof. R.Kulikowski and Prof. J.Hołubiec from the Systems Research Institute of the Polish Academy of Sciences, where this book was written, is gratefully acknowledged. Finally, the author wishes to thank Mrs. I.Forowicz and Mrs. E.Grudzińska for patiently typing the manuscript.

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