

# Lecture Notes in Control and Information Sciences

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J. Zarzycki

Nonlinear Prediction  
Ladder-Filters for Higher-Order  
Stochastic Sequences

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## PREFACE

In this work we shall be concerned with the problem of nonlinear least-squares prediction of higher-order stochastic sequences using nonlinear orthogonal digital filters. The nonlinear problem will be considered as a generalization of the linear least-squares prediction problem.

The linear least-squares estimation theory is associated with second-order stochastic sequences, for which the orthogonal prediction or innovations linear filters (producing white noise when driven by the given sequence) as well as modeling or shaping filters (whose output is statistically equivalent to the given sequence, when driven by white noise) can be established.

In practice, the linear orthogonal filters can be computed via recursive procedures, in which one will not have to recompute the whole filter each time the permitted complexity is increased. Hence, the same idea underlies both the ladder-structures implemented in modern digital filters theory, and the theory of orthogonal (Fourier) expansions in Hilbert spaces. One of the most important property of the orthogonal digital filter is preservation of 'energy' which assures inherent numerical stability of the filter. A remarkable result of this theory is that any transfer function can be realized by means of an orthogonal filter whose modular structure can be implemented using sophisticated 'building-blocks' with VLSI integrated circuits (namely CORDICS processors).

The linear theory results in the optimum (least-squares) stochastic approximation of second-order sequences. Therefore, the linear estimation

filter becomes the best possible filter for a Gaussian sequence (whose properties are completely characterized by the second-order statistics). If the underlying sequence is non-Gaussian, the linear estimation accuracy may be not satisfactory. In that case, a nonlinear approach to the problem should be introduced in order to improve the accuracy.

In this work we wish to present efficient algorithms of nonlinear least-squares prediction filters for higher-order stochastic sequences, resulting in the optimum approximate nonlinear digital filters of the Volterra-Wiener class. These nonlinear ladder-filters will generalize the linear filters, preserving most of their properties (orthogonality and modular realizations, among others), and yielding better estimation accuracy for higher-order (and non-Gaussian) stochastic sequences.

We will mention here only those papers which are closely connected to the subject of this work, referring for more complete bibliography to the papers cited (and the references therein).

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CONTENTS

CHAPTER 1, INTRODUCTION .....	1
CHAPTER 2. NONLINEAR PREDICTION FILTER PROBLEM; A UNIFIED APPROACH .....	13
2.1 Higher-order stochastic sequences .....	13
2.2 Nonlinear least-squares prediction; Algebraic approach.....	20
2.3 Nonlinear least-squares prediction; Geometric approach.....	24
2.3.1 Space of the regular Volterra functional polynomials.....	24
2.3.2 Space of generalized coefficient-matrices.....	26
2.3.3 Space of generalized z-polynomials.....	29
2.3.4 Isometries.....	32
2.3.5 Stochastic nonlinear estimation.....	35
2.3.6 Optimum generalized matrix approximation.....	36
2.3.7 Optimum generalized polynomial approximation.....	38
CHAPTER 3, GENERALIZED NONLINEAR LADDER-FILTERS.....	40
3.1 Index-sets and their ordering.....	41
3.2 Nonlinear filter algorithm; time-domain approach.....	48
3.2.1 'Local' estimates and errors.....	51
3.2.2 Decomposition of subspaces.....	55
3.2.3 Orthonormal bases.....	57
3.2.4 Generalized Cholesky factorizations.....	60
3.2.5 M-D Fourier series expansion.....	61
3.2.6 Order-update recursions.....	62
3.2.7 Optimum approximation of the M-D impulse responses.....	70
3.2.8 Estimation accuracy.....	71
3.3 Nonlinear filter algorithm; transform-domain approach.....	74
3.3.1 'Local' estimates and errors.....	76
3.3.2 Decomposition of subspaces, ON bases and M-D Fourier expansion.....	79
3.3.3 Order-update recursions.....	83
3.3.4 Optimum ON approximation of the set of M-D transfer functions.....	85
3.4 Nonlinear time-variant ladder-filter.....	86
CHAPTER 4, TIME-INVARIANT AND 'QUASI-LINEAR' LADDER-FILTERS.....	91
4.1 Shift-invariance of inner-products.....	91
4.2 Time-invariant nonlinear ladder-filter algorithm.....	93
4.3 'Quasi-linear' ladder-filters.....	98
4.4 Experimental example.....	106
CONCLUDING REMARKS .....	109
REFERENCES .....	110
APPENDIX 1 .....	116
APPENDIX 2 .....	127