Lecture Notes
in Control and Information Sciences 201

Editor: M. Thoma

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Robust Stability and Convexity

An Introduction

Springer-Verlag
London Berlin Heidelberg New York
Paris Tokyo Hong Kong
Barcelona Budapest

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ISBN 3-540-19919-5 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-19919-5 Springer-Verlag New York Berlin Heidelberg

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Typesetting: Camera ready by author
Printed and bound at the Athenæum Press Ltd, Gateshead
69/3830-543210 Printed on acid-free paper

To the memory of Josef Kogan

Preface

A fundamental problem in control theory is concerned with stability of a given linear system. The system designer often wants to know if all the roots of the systems characteristic polynomial are located in a pre–specified open region of the complex plane (the left half plane, and the unit disc are important examples of such regions). In many applications the coefficients of the characteristic polynomial are functions of independent physical parameters (as, for example, coefficients of friction, spring constants, masses, capacitances, inductances, etc.). The design of a control system is generally based on a simplified model, and the true values of the physical parameters may differ from the assumed values. Consequently, it is of interest to verify root location of the entire *family* of characteristic polynomials.

When regions of stability of dynamic systems in the parameter space are described by complicated expressions it is difficult to visualize the size and the shape of these regions. An attractive way to resolve the problem is to embed a simple geometric figure inside the region. In many practical situations the polynomial family can be associated with the n dimensional box $\mathbf{B}^n = \{\mathbf{x} : 0 \leq x_i \leq 1\}$, while each frequency w is associated with a function f that maps the box into the complex plane $\mathbf{C}$. The verification of stability boils down to the following mathematical problem: Determine whether the boundary of the image of the box $\partial f(\mathbf{B}^n)$ contains the origin in the complex plane.

This problem can be reduced to a constrained optimization problem. For affine functions f the optimization problem admits an analytic solution. For nonlinear functions f there exist no analytic solutions, and associated numerical calculations are computationally difficult and time consuming.

In this manuscript we introduce and describe a set of *principal points* $\mathbf{X}_\mathrm{P}$. The set of principal points is a subset of the box whose image under f covers the boundary of the image of the box. When f is an affine mapping in order to describe the boundary of $f(\mathbf{B}^n)$ one has to select a finite set of *principal vertices.* For example, a striking fact discovered by Kharitonov [Kh] shows that when coefficients of polynomials of degree n vary independently in pre–specified intervals there exist **four** principal vertices, and the interval polynomial family is Hurwitz stable if and only if the polynomials associated with the principal vertices are Hurwitz stable. This remarkable **four** vertices result holds **independent** of n! The set of the principal vertices is, in general, frequency dependent. Generically in the nonlinear case the set of principal points consists of finitely many *one* dimensional manifolds (which are sometimes just linear segments). As a rule the manifolds are frequency dependent, however in Section 4.4 we provide an example of a nonlinear system with frequency independent principal linear segments.

The principal points approach can be traced to the works of Zeheb and Walach [ZW], and Zeheb [Z]. Systematic applications of the approach generate new simple proofs of many known robust stability results, and lead to necessary and sufficient robust stability conditions for polynomial families with coefficients depending multiaffinely on parameters, and quasipolynomial families with uncertainties in coefficients and delays. On the other hand an application of the principal points approach to Hurwitz stability of box polynomials recovers the four vertices result (see Section 2.8).

Although the main motivation for the study of root location of characteristic functions comes from related stability problems of linear systems it is important to distinguish between the two problems. We address this issue in detail in Chapter 5.

ACKNOWLEDGMENT

I express my appreciation to Ya.Z. Tsypkin whose seminar on robust stability held in George Washington University triggered my interest in the subject. While preparing notes and reference material for a course on robust stability in the summer of 92 I was encouraged by Professor Tsypkin to integrate the material into a book.

During the last few years I have been fortunate to collaborate with N. Cohen, J. Hocherman, V.L. Kharitonov, A. Leizarowitz, B.T. Polyak, M. Teboulle, and E. Zeheb. Many of the new research results reported in this manuscript are an outgrowth of this collaboration. Many helpful suggestions and useful discussions with B.R. Barmish, F.G. Boese, N.K. Bose, M.S. Gowda, E.I. Jury, B.Ya. Kogan, D.D. Šiljak, and R. Tempo helped in building my knowledge of system theory, and many subjects related to robust stability.

My thanks also go to students at University of Maryland Baltimore County who participated in a course based on early versions of the manuscript. I would like particularly mention R. Sznajder whose remarks led to an elegant proof of Tsypkin–Polyak stability criterion given in Section 3.5.

Last but not least, I would like to express my thanks to R. Rostamian and W. Shyong for their help with computer related issues.

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