

# Lecture Notes in Mathematics

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Jürgen Fischer

An Approach to the  
Selberg Trace Formula  
via the Selberg Zeta-Function

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**Author**

Jürgen Fischer  
Mathematisches Institut  
Einsteinstraße 62, 4400 Münster, Federal Republic of Germany

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