

# Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

1326

---

P.S. Landweber (Ed.)

## Elliptic Curves and Modular Forms in Algebraic Topology

Proceedings of a Conference held at  
the Institute for Advanced Study  
Princeton, Sept. 15–17, 1986

---



Springer-Verlag

Berlin Heidelberg New York London Paris Tokyo

**Editor**

Peter S. Landweber  
Department of Mathematics, Rutgers University  
New Brunswick, NJ 08903, USA

Mathematics Subject Classification (1980): 11F 11, 33A 25, 33A 45, 55N 22,  
57S 15, 81E 99

ISBN 3-540-19490-8 Springer-Verlag Berlin Heidelberg New York  
ISBN 0-387-19490-8 Springer-Verlag New York Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. Duplication of this publication or parts thereof is only permitted under the provisions of the German Copyright Law of September 9, 1965, in its version of June 24, 1985, and a copyright fee must always be paid. Violations fall under the prosecution act of the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1988  
Printed in Germany

Printing and binding: Druckhaus Beltz, Hemsbach/Bergstr.  
2146/3140-543210

## Preface

This volume contains the proceedings of a conference held September 15-17, 1986 at the Institute for Advanced Study in Princeton, New Jersey.

The introductory article provides an account of the recent history of the field of elliptic genera and elliptic cohomology, the central theme of the conference.

The main surprise at the conference was that its original conception was too narrow, and that geometry and physics also enter prominently into this area. For this, see the paper by Ed Witten.

I am grateful to Noriko Yui for permitting her paper on the formal groups of Jacobi quartics, an especially relevant topic for the study of elliptic genera, to be included in this volume.

Thanks are due to David and Gregory Chudnovsky for the suggestion to hold such a conference, and to Bob Stong for substantial advice throughout. It is also a pleasure to thank the School of Mathematics at the Institute for Advanced study, for providing the setting for the conference, and especially Linda Sheldon for much aid. Partial financial support was provided by the National Science Foundation.

Conference Talks

- S. Ochanine, Elliptic genera for  $S^1$  manifolds
- P. Landweber, Periodic cohomology theories defined by elliptic curves
- D. Chudnovsky and G. Chudnovsky, Elliptic formal groups over  $\mathbb{Z}$  and  $\mathbb{F}_p$  in applications to topology, number theory and computer science
- R. Stong, Dirichlet series and homology theories
- D. Ravenel, BP-theory for number theorists
- M. Hopkins, Characters and generalized cohomology
- J. Morava, The Weil group as automorphisms of the extraordinary K-theories
- D. Zagier, Modular forms, elliptic functions, Jacobi forms
- E. Witten, Elliptic genera and quantum field theory
- J. Lepowsky, Infinite dimensional algebras and modular functions
- J. Stasheff, Homotopical Lie representations in theoretical physics

Table of Contents

P. S. LANDWEBER, Elliptic Genera: an Introductory Overview .....	1
D. V. CHUDNOVSKY and G. V. CHUDNOVSKY, Elliptic Formal Groups over $\mathbb{Z}$ and $\mathbb{F}_p$ in Applications to Number Theory, Computer Science and Topology .....	11
P. S. LANDWEBER, Elliptic Cohomology and Modular Forms .....	55
P. S. LANDWEBER, Supersingular Elliptic Curves and Congruences for Legendre Polynomials .....	69
J. MORAVA, Some Weil Group Representations Motivated by Algebraic Topology .....	94
S. OCHANINE, Genres Elliptiques Equivariants .....	107
D. C. RAVENEL, Complex Cobordism Theory for Number Theorists .....	123
L. SMITH and R. E. STONG, Dirichlet Series and Homology Theories .....	134
J. D. STASHEFF, Constrained Hamiltonians: An Introduction to Homological Algebra in Field Theoretical Physics .....	150
E. WITTEN, The Index of the Dirac Operator in Loop Space .....	161
N. YUI, Jacobi Quartics, Legendre Polynomials and Formal Groups ...	182
D. ZAGIER, Note on the Landweber–Stong Elliptic Genus .....	216