

Lecture Notes in Physics

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220

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Effective Lagrangians in
Quantum Electrodynamics



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PREFACE

With these notes we would like to provide an introduction to the subject of effective Lagrangians for quantum electrodynamics (QED). Although this topic is interesting in its own right, it also provides us with an example of several important calculational techniques of QED which are usually not found in standard texts. Moreover, studying vacuum problems in QED, where matters are fairly well understood, can be a helpful preparation for similar considerations in quantum chromodynamics and other more complicated theories. In contrast to the latter, electrodynamics has the advantage that many calculations can still be done analytically.

To make our computations as transparent as possible, many of them are presented in great detail. In particular, this is true for the 2nd, 3rd and 4th sections. The reader who is mainly interested in general concepts could omit the rather technical derivations of these sections in a first reading.

We wish to thank Christel Kienle for her endless patience and skill in typing the various versions of the manuscript.

Tübingen, September 1984

W. Dittrich

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TABLE OF CONTENTS

(1) Introduction	1
(2) The Electron Propagator in a Constant External Magnetic Field	28
(3) The Mass Operator in a Constant External Magnetic Field	37
(4) The Polarization Tensor in a Constant External Magnetic Field	56
(5) One-Loop Effective Lagrangian	73
(6) The Zeta Function	98
(7) Two-Loop Effective Lagrangian	121
(8) Renormalization Group Equations	147
(9) Applications and Discussion	167

APPENDIX

(A) Units, Metric, Gamma Matrices	183
(B) One-Loop Effective Lagrangian of Scalar QED	186
(C) The Casimir Effect	197
(D) Derivatives of $W[A]$	206
(E) Power Series and Laurent Series of $K(z,v)$	210
(F) Contact Term Determination in Source Theory	228
(G) One-Loop Effective Lagrangian as Perturbation Series	232
(H) Summary of the Most Important Formulae	235

REFERENCES

242