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Preface

These notes are based on a series of lectures I gave in Lausanne and Geneva in January and February of 1981 in the framework of the "Troisième Cycle de la Physique en Suisse Romande". A preliminary version of these notes was preprinted by the "Troisième Cycle". Compared to that version numerous changes and corrections have been made. In particular an erroneous proof of the combinatorial Corollary 3.9 has been replaced by a simpler and (I hope) correct one. I am grateful to many people, in particular Christian Borgs, Krzysztof Gawędzki, Konrad Osterwalder and Simon Ruijsenaars for pointing out various errors in the earlier version and for suggesting some improvements. I am also much indebted to Ricardo Neves da Silva for his help in preparing the manuscript.

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