Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

931

David M. Arnold

Finite Rank Torsion Free Abelian Groups and Rings

Springer-Verlag
Berlin Heidelberg New York 1982

Author
David M. Arnold
Department of Mathematical Sciences
New Mexico State University
Las Cruces, NM 88003, USA

AMS Subject Classifications (1980): 20K15, 20K30, 20K40

ISBN 3-540-11557-9 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-11557-9 Springer-Verlag New York Heidelberg Berlin

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.
2141/3140-543210

INTRODUCTION

These notes contain a largely expository introduction to the theory of finite rank torsion free abelian groups developed since the publication of "Infinite Abelian Groups," Vol. II, L. Fuchs, in 1973. As reflected in Chapter XIII of that text, the subject consists of a satisfactory theory for direct sums of rank 1 groups due to R. Baer in 1937; a uniqueness of quasi-direct sum decompositions up to quasi-isomorphism due to B. Jónsson in 1959; a realization of subrings of finite dimensional Q-algebras as endomorphism rings due to A.L.S. Corner in 1963; a variety of pathological direct sum decompositions; and some apparently miscellaneous results largely relegated to the exercises.

Substantial progress has been made in the subject since 1973. Most notable are the stable range conditions proved by R.B. Warfield, near isomorphism as introduced by E.L. Lady, and the application of properties of subrings of finite dimensional Q-algebras to finite rank torsion free abelian groups via a Morita-like duality developed by E.L. Lady and the author. Consequently, some older results of R. Beaumont, R. Pierce, and J. Reid (c. 1960) involving subrings of finite dimensional Q-algebras gain new importance. Thus a systematic introduction to the theory of finite rank torsion free abelian groups and subrings of finite dimensional Q-algebras seems timely.

The theory of direct sums of rank-1 torsion free abelian groups has been combined with the theory of totally projective groups to characterize a class of mixed abelian groups (Warfield [7] and Hunter-Richman [1]). The category Walk, as discussed in Warfield [7], has been used to investigate mixed abelian groups. A secondary goal of these notes is to survey the known results for finite rank torsion free abelian groups with an eye towards eventual application to mixed groups of finite torsion free rank via the category Walk. Some progress along these lines is reported by Warfield [7]. Other potential applications include the study of mixed abelian groups of finite torsion free rank via the category Warf, as discussed in Arnold - Hunter - Richman [1], and valuated finite direct sums of torsion free cyclic groups, as discussed in a series of papers by E. Walker, F. Richman, R. Hunter, and the author. In particular, Rotman [1] shows that finite rank torsion free groups are characterized in terms of valuated finite direct sums of torsion free cyclic groups.

These notes were developed for a graduate course taught by the author as part of the Year of Algebra at the University of Connecticut during academic year 1978-1979. The students were assumed to have had a graduate course in algebra (fundamental concepts and classical theory of artinian rings are given in Section 0 and the exercises) but little or no exposure to finite rank torsion free abelian groups or subrings of finite dimensional Q-algebras.

Except for portions of Sections 0, 1, and 2 there is little overlap with the results proved in Fuchs [7], Vol. II. There are exercises at the end of each section, some of which are contributed by others as noted, devoted to an extension and elaboration of the results presented or of the requisite background material. No attempt has been made to state or prove results in maximum generality, but in most cases references are given for more general theorems.

Sections 1-4 include a classical introduction to the subject of finite rank torsion free abelian groups as well as some generalizations of type and applications (Richman [1] and Warfield [1]) in Section 1; properties of rank-2 groups in terms of their typeset (Beaumont-Pierce [2]) in Section 3; and characterizations of pure subgroups of finite rank completely decomposable groups (Butler [1]) in Section 4.

Generalizations of such topics as finite rank completely decomposable groups and Baer's Lemma are developed in Sections 5-6 as derived by Arnold-Lady [1] and Arnold-Hunter-Richman [1].

Section 7 includes a proof of the Krull-Schmidt Theorem in additive categories with Jónsson's quasi-decomposition theorem and some essential properties of near isomorphism, due to Lady [1], as corollaries.

Stable range conditions are considered in Section 8 (Warfield [5]) as well as cancellation and substitution properties (Warfield [5], Fuchs-Loonstra [2], and Arnold-Lady [1]), exchange properties (Warfield [5], Monk [1], Crawley-Jónsson [1]) and self-cancellation (Arnold [7]).

Sections 9-11 include an extensive introduction to the subject of subrings of finite dimensional Q-algebras, including a proof of the Jordan-Zassenhaus Theorem for Z-orders, derived in part from Reiner [1] and Swan-Evans [1]. The fact that the additive groups of such rings are finite rank torsion free is exploited to avoid completions in the derivation of the theory. Moreover, localization at primes of Z is consistently used instead of localization at prime ideals of more general domains.

The relationship between near isomorphism and genus class of lattices over orders is examined in Section 12. Classical properties of genus classes of lattices over orders are derived and used to develop properties of near isomorphism of finite rank torsion free abelian groups.

The structure of Grothendieck groups of finite rank torsion free abelian groups is considered in Section 13, as developed by Lady [2] and Rotman [2].

Section 14 includes characterizations of additive groups of subrings of finite dimensional Q-algebras, due to Beaumont-Pierce [1] and [3], including a proof of the Wedderburn principal theorem and a simplified proof of the analog for subrings of finite dimensional Q-algebras.

Several classes of groups are given in Section 15, providing an appropriate setting for the development of Murley groups (Murley [1]) and strongly homogeneous groups (Arnold [6]).

The author expresses his appreciation to the University of Connecticut Department of Mathematics for the invitation to participate in the Year of Algebra. Special thanks are due to R. Pierce, J. Reid, C. Vinsonhaler, W. Wickless,

R. Weigand, S. Weigand, T. Faticoni, and J. Chung for their encouragement, assistance, and extraordinary patience with the development of these notes. The author is also indebted to E.L. Lady for the opportunity to read an unpublished set of lecture notes on the subject, R. B. Warfield, Jr. for numerous discussions regarding the contents of these notes, and D. Boyer, A. Mader, M. Maher, K. Rangaswamy, T. Giovannitti, J. Moore, U. Albrecht and J. Stelzer for their helpful comments and suggestions. A special note of thanks is due to Lorna Schriver for her accurate and efficient typing of these notes as well as her remarkable ability to interpret the author's frequently illegible handwriting. Finally, any errors that remain are directly attributable to the author.

TABLE OF CONTENTS