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Bernard Dacorogna

Weak Continuity
and Weak Lower Semicontinuity
of Non-Linear Functionals



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PREFACE

These notes are the result of a graduate course given at Brown during the first quarter of 1981. They should be considered as an introduction to the subject. They are not intended to be a complete presentation of all the results in this area. The results presented here are not all new and obviously a large part of the first and second chapter owes much to various works of F. Murat and L. Tartar on compensated compactness.

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B. Dacorogna
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by

B. Dacorogna

ABSTRACT

These notes deal with the behavior of nonlinear functionals with respect to weak convergence. In the first chapter we investigate several necessary and sufficient conditions in order that a nonlinear function is weakly continuous or weakly lower semicontinuous. In Chapter II we give some applications of the results of Chapter I to partial differential equations and to nonlinear elasticity. In the last chapter we deal with dual and relaxed variational problems.

TABLE OF CONTENTS

	Page
Introduction.....	1
Chapter I. Compensated Compactness	
§1. Preliminary Result (Case without Assumptions on the Derivatives).....	7
§2. Case with Assumptions on the Derivatives.....	11
§3. Legendre-Hadamard Condition and other Necessary Conditions.	19
§4. The Quadratic Case and Some Generalizations.....	31
§5. An Important Example: The Variational Case.....	39
§6. Parametrized Measures.....	52
Chapter II. Applications	
§1. Nonlinear Conservation Laws.....	59
§2. Existence Theorems in Nonlinear Elasticity.....	68
Chapter III. Dual and Relaxed Problems	
§1. Dual Problems.....	74
§2. Relaxed Variational Problems and Applications.....	80
Appendix.....	100
References.....	113
Index	117