Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

1087

Władysław Narkiewicz

Uniform Distribution of Sequences of Integers in Residue Classes

Springer-Verlag
Berlin Heidelberg New York Tokyo 1984

Author

Władysław Narkiewicz
Wrocław University, Department of Mathematics
Plac Grunwaldzki 2–4, 50-384 Wrocław, Poland

AMS Subject Classification (1980): 10A35, 10D23, 10H20, 10H25, 10L20, 10M05

ISBN 3-540-13872-2 Springer-Verlag Berlin Heidelberg New York Tokyo
ISBN 0-387-13872-2 Springer-Verlag New York Heidelberg Berlin Tokyo

Printing and binding: Beltz Offsetdruck, Hemsbach / Bergstr.
2146/3140-543210

To my teacher
Professor Stanisław Hartman
on his seventieth anniversary

INTRODUCTION

The aim of these notes, which form an extended version of lectures given by the author at various places, is to present a survey of what is known about uniform distribution of sequences of integers in residue classes. Such sequences were studied since the beginning of this century, when L.E.Dickson in his Ph.D. thesis made a thorough study of permutational polynomials, i.e. polynomials inducing a permutation of residue classes with respect to a fixed prime.

We shall also consider weak uniform distribution of sequences, meaning by that uniform distribution in residue classes (mod N), prime to N. The standard example here is the sequence of all primes, which is weakly uniformly distributed (mod N) for every integer N.

After proving, in the first chapter, certain general results we shall consider uniform distribution of certain types of sequences, starting with polynomial sequences and considering also linear recurrent sequences and sequences defined by values of additive arithmetical functions. This will be done in chapter II-IV. In the last two chapter we shall study uniform distribution of sequences defined by multiplicative functions, in particular those, which are "polynomial-like", i.e. satisfy the condition $f(p^k) = P_k(p)$ for primes p and $k \geq 1$ with suitable polynomials $P_1, P_2, \ldots$. In particular we shall consider classical arithmetical functions, like the number or sum of divisors, Euler's φ-function and Ramanujan's τ-function. This will lead to certain questions concerning the value distribution of polynomials.

Our tools belong to the classical number theory and include fundamentals of the theory of algebraic numbers. In certain places we shall use more recent work, like the theorems of P.Deligne, J.P.Serre and H.P.F.Swinnerton-Dyer on modular forms, which will be used in the study of Ramanujan's function. In such cases we shall explicitly state the result needed with a proper reference.

We shall use notation which is standard in number theory. In particular we shall denote the number of divisors of n by $d(n)$, $\sigma(n)$ will

denote the sum of divisors of n and $\sigma_k(n)$ the sum of their k-th powers, only positive divisors being taken into account. The cardinality of a set A will be denotes by $\sharp A$ and the letter p will be reserved for primes (except when inside a word). The ring of integers will be denoted by Z and $G(N)$ will be the multiplicative group of restricted residue classes (mod N), i.e. the group of invertible elements of the factor ring Z/NZ. Theorems, lemmas and propositions will be consecutively numbered in each chapter. Certain open problems will be stated in the text, numbered consecutively through all chapters.

I wish to express my gratitude to Mrs Dambiec from the Department of Mathematics of Wrocław University for the patient and careful preparation of the typescript.

Wrocław, February 1984

CONTENTS