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# Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

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## Brauer Groups in Ring Theory and Algebraic Geometry

Proceedings, University of Antwerp U.I.A.,  
Belgium, August 17 – 28, 1981

Edited by F. van Oystaeyen and A. Verschoren

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Springer-Verlag  
Berlin Heidelberg New York 1982

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AMS Subject Classifications (1980): 14 F 05, 16 A 16, 13 A 20, 14 A 15

ISBN 3-540-11216-2 Springer-Verlag Berlin Heidelberg New York

ISBN 0-387-11216-2 Springer-Verlag New York Heidelberg Berlin

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Printed in Germany

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.

2141/3140-543210

## FOREWORD

The workshop "Brauer groups in Geometry and Ring Theory", organized at the University of Antwerp, U.I.A. in August 1981, has been financially supported by the Belgian Foundation for Scientific Research N.F.W.O. and by U.I.A. We thank both institutions for their continuous support.

Allow us to extend the informality of this meeting to this foreword, so we will just use these few words to thank everybody who contributed to the workshop : participants, lecturers and the local committee.

The papers in these proceedings have been ordered so that the first part contains the papers of a more ringtheoretical nature, while the more geometrically inspired papers are in the second part. It is left to the reader to decide where the first part ends and the second starts.

F. Van Dystaeyen

A. Verschoren

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