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Lie Group Representations II

Proceedings of the Special Year
held at the University of Maryland, College Park 1982–1983

Edited by R. Herb, S. Kudla, R. Lipsman and J. Rosenberg



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PREFACE

The Department of Mathematics of the University of Maryland conducted a Special Year in Lie Group Representations during the academic year 1982-1983. This volume is the second (of three) containing articles submitted by the main speakers during the Special Year. Most of the invited speakers submitted articles, and virtually all of those appearing here deal with the subject matter on which the authors lectured during their visits to Maryland.

The Special Year program at Maryland represents a thriving departmental tradition—this being the fourteenth consecutive year in which such an event has taken place. As usual, the subject matter was chosen on the basis of active current research and the interests of departmental members. The modern theory of Lie Group Representations is a vast subject. In order to keep the program within bounds, the Special Year was planned around five distinct intensive periods of activity—each one (of three weeks duration) devoted to one of the main branches of current research in the subject. During those periods (approximately) eight distinguished researchers were invited to present lecture series on areas of current interest. Each visitor spent 1-3 weeks in the department and gave 2-5 lectures. In addition, during each period approximately 8-10 other visitors received financial support in order to attend and participate in the Special Year activities. Thus each period had to some extent the flavor of a mini-conference; but the length of the periods, the fact that visitors were provided with office space and the (relatively) low number of lectures per day also left ample time for private discussion and created the atmosphere of "departmental visitor" rather than "conference participant." Furthermore, as part of the Special Year the department was fortunate to have in residence D. Barbasch, J. Bernstein and J.-L. Brylinski for the Fall 1982 semester, and B. Blank for the Spring 1983 semester. These visitors ran semester-long seminars in Group Representations.

All of the activities of the Special Year were enthusiastically supported by the department, its faculty and graduate students.

Although most of the cost of the Special Year was borne by the department, the NSF did provide a generous amount of supplementary support. In particular, the contributions to the additional visitors were entirely funded by NSF. The Mathematics Department is grateful to the Foundation for its support of the Special Year. The Organizing Committee would also like to express its gratitude to the Department for its support. In particular the splendid efforts of Professors W. Kirwan, J. Osborn, G. Lehner, as well as of N. Lindley, D. Kennedy, D. Forbes, M. Keimig, and J. Cooper were vital to the success of the Special Year. The outstanding job of preparation of manuscripts by Berta Casanova, June Slack, Anne Eberly and Pat Pasternack, was of immense help in producing this volume so quickly. Also we are grateful to Springer-Verlag for its cooperation. Finally we are very pleased that so many of our participants provided us with high quality manuscripts, neatly prepared and submitted on time. It is our conviction that the theory of Group Representations has profited greatly from the efforts of all the above people towards the Special Year.

The Editors
August 1983

INTRODUCTION

We have made a serious attempt to group the papers (within the three volumes) according to the Periods in which they were presented and according to subject matter. However we were also influenced by the time at which manuscripts became available, and by a desire to equalize the size of the volumes. This (second) volume contains papers from Period II of the Special Year. The program for that period was as follows:

PERIOD II. The Langlands Program—Arithmetic Groups, Automorphic Representations, Automorphic Forms, L-Groups, Base Change, Orbital Integrals, Adelic and Local Theory

- J. Arthur -- The Trace formula and lifting problems
- J. Bernstein -- P-Invariant distributions on $GL(n)$
- W. Casselman -- Paley-Wiener theorems, automorphic forms, and cohomology
- S. Gelbart -- Automorphic forms on unitary groups
- R. Howe -- On a question of Weil: Dual pairs and the poles of L-functions
- H. Jacquet -- Residual spectrum, especially for $GL(n)$
- D. Kazhdan -- Harmonic analysis and orbital integrals for p-adic groups
- R. Langlands -- Orbital integrals for p-adic groups
- I. Piatetski-Shapiro -- L-fonctions for $GSp(4)$

Although Paul Sally delivered his lecture series during Period III (see volume I for the program), he was present during Period II, and since his topic fit more closely with those presented here his paper is included in this volume.

The additional participants during Period II of the Special Year were

- | | |
|---------------|-------------|
| L. Clozel | J. Rogawski |
| Y. Flicker | P. Sally |
| P. Kutzko | F. Shahidi |
| J.-P. Labesse | J. Shalika |
| A. Moy | B. Speh |
| S. Rallis | A. Terras |
| F. Rodier | J. Tunnell |

SPECIAL YEAR DATA

A. The five periods of activity of the Special Year were as follows:

- I. Algebraic Aspects of Semisimple Theory -- Sept. 7, 1982 - Oct. 1, 1982
- II. The Langlands Program -- Nov. 1, 1982 - Nov. 19, 1982
- III. Analytic Aspects of Semisimple Theory -- Jan. 24, 1983 - Feb. 11, 1983.
- IV. The Orbit Method -- Feb. 28, 1983 - March 18, 1983
- V. Applications -- April 18, 1983 - May 6, 1983

B. The speakers and the dates of their visits were:

Period I

Thomas Enright, UCSD (9/7 - 9/22)
Anthony Joseph, Weizmann Institute (9/21 - 9/25)
Bertram Kostant, MIT (9/7 - 9/14)
George Lusztig, MIT (9/7 - 9/11)
Wilfried Schmid, Harvard (9/13 - 9/18)
David Vogan, MIT (9/27 - 10/1)
Nolan Wallach, Rutgers (9/20 - 10/1)

Period II

James Arthur, Toronto (11/1 - 11/19)
William Casselman, British Columbia (11/3 - 11/12)
Stephen Gelbart, Cornell (11/1 - 11/12)
Roger Howe, Yale (11/8 - 11/12)
Hervé Jacquet, Columbia (11/1 - 11/12)
David Kazhdan, Harvard (11/1 - 11/12)
Robert Langlands, IAS (11/1 - 11/12)
Ilya Piatetski-Shapiro, Yale (11/1 - 11/12)

Period III

Mogens Flensted-Jensen, Copenhagen (1/24 - 2/11)
Sigurdur Helgason, MIT (1/24 - 1/28)
Anthony Knapp, Cornell (2/2 - 2/4)
Paul Sally, Chicago (1/24 - 2/11)
V. S. Varadarajan, UCLA (1/24 - 2/11)
Garth Warner, Washington (2/7 - 2/8)
Gregg Zuckerman, Yale (1/24 - 2/4)

Period IV

Lawrence Corwin, Rutgers (3/7 - 3/11)
Michael Cowling, Genova (3/2 - 3/4)
Michel Duflo, Paris, (2/28 - 3/11)
Roger Howe, Yale (3/7 - 3/11)
Henri Moscovici, Ohio State (3/7 - 3/18)
Richard Penney, Purdue (3/7 - 3/11)
Lajos Pukanszky, Penn (3/7, 3/11 - 3/18)
Wulf Rossmann, Ottawa (2/28 - 3/4)
Michèle Vergne, MIT (3/3 - 3/15)

Period V

Lawrence Corwin, Rutgers (4/18 - 4/29)
Bernard Helffer, Nantes (4/18 - 5/6)
Sigurdur Helgason, MIT (4/18 - 4/22)
Roger Howe, Yale (4/18 - 4/22)
Adam Koranyi, Washington Univ. (4/18 - 5/6)
Henri Moscovici, Ohio State (4/25 - 4/30)
Richard Penney, Purdue (4/25 - 5/6)
Linda Rothschild, Wisconsin (4/18 - 4/22)

C. The Organizing Committee for the 1982-1983 Special Year in Lie Group Representations is

Rebecca Herb
Raymond Johnson
Stephen Kudla
Ronald Lipsman (Chairman)
Jonathan Rosenberg

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* For papers with more than one author, an asterisk indicates the author who delivered the lectures.