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Non-linear Partial Differential Operators and Quantization Procedures

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Edited by S. I. Andersson and H.-D. Doebner

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PREFACE

Non-linear physical systems and their mathematical structure form one of the most active fields in present mathematics and mathematical physics. This volume covers parts of that topic. It reports on differential geometrical and topological properties of those non-linear systems, which can be viewed physically as models for quantized non-relativistic particles constrained, i.e. localized, on a (smooth) manifold or as classical or quantized fields with non-linear field equations. The contributions of this volume show how to deal with these different types of non-linearities. There are various physically motivated approaches to both of them. For systems constrained on a manifold generically geometric methods are used with promising mathematical and physical results. Now that the feeling has dissipated, that global solutions of non-linear field equations are"extra-terrestrial beasts" (see the contribution of I.E. SEGAL), also here a more global and geometrical approach is applied with extreme success, we refer e.g. to the application of twistor geometry or to the analysis of solution manifolds of non-linear equations. The structures of both types of non-linearities are deeply related.

A summer workshop in connection with the above programme was held in July 1981 at the Technical University in Clausthal, Institute for Theoretical Physics and an international conference on mathematical physics was organized parallel to the workshop. The lectures at the workshop and some of the contributions to the conference are collected and edited in an updated version in this volume.

Quantization Procedures

Quantizations of non-relativistic (mechanical) systems constrained on a smooth manifold are discussed. The method of geometrical quantization is justified on more physical grounds and presented in a new context by R.J. BLATTNER. The kinematics of such systems is described with the notion of a "quantized Borel kinematics" without using the phase space and its symplectic structure by B. ANGERMANN, H.D. DOEBNER and J. TOLAR. A method for the quantization of constrained systems is proposed by J. SNIATICKY and is based on aspects of Dirac's theory and on a reduced phase space. The late S. PANEITZ defined "stable subvarieties" of so-

lution manifolds of a class of time dependent Hamiltonian systems and "stable polarizations" and shows how these notions apply to certain systems with non-linear scattering. The Frobenius reciprocity theorem is discussed by V. GUILLEMIN and S. STERNBERG from the symplectic point of view and is linked to some structures of the geometric quantization method and to induced representations of symmetry groups.

Non-Linear Field Equations

The general properties of solution "manifolds" of non-linear field equations are discussed. I.E. SEGAL reviews authoritatively historical aspects and part of the present status of this field. R.O. WELLS describes with details and applications the twistor geometric approach to classical field equations. One of the physically interesting non-linear systems with a genuine geometry is the non-linear sigma model. A comprehensive report on this model is given by M. FORGER. F.B. PASEMANN describes a quantization of gauge theories based on their geometrical structure as Kaluza-Klein theories on a principle bundle and on de Rham - p - currents as fields and potentials.

From the Clausthal workshop and Conference on "Non-Linear Partial Differential Operators and Quantization Procedures" this volume contains only part of the lectures presented there. The editors agree with the general editorial requirements that a lecture notes volume should be homogenous and that papers presenting mainly already known results or having the character of a research announcement should not be included. Some manuscripts were not received in time. The articles in Part I and II are arranged in alphabetical order.

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The Editors.

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