

Lecture Notes in Mathematics

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Regular Boundary Value Problems
Associated with Pairs of
Ordinary Differential Expressions



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Preface

Numerous papers have been devoted to the study of eigenvalue problems associated with pairs L, M of ordinary differential operators. They concern the solutions f of $Lf = \lambda Mf$ subject to boundary conditions. In an earlier paper [9] we showed how these problems have a natural setting within the framework of subspaces in the direct sum of Hilbert spaces. In these notes we work out in detail the regular case, where the coefficients of the operators L and M are nice on a closed bounded interval \bar{I} and M is assumed to be positive definite, in the sense that $(Mf, f)_2 \geq c^2(f, f)_2$, $f \in C_0^\infty(\bar{I})$, for some constant $c > 0$. It is hoped that this detailed knowledge of the regular case will lead to a greater understanding of the more involved singular case, where L and M are defined on an arbitrary, possibly unbounded, open interval.

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