

# Lecture Notes in Control and Information Sciences

Edited by A.V. Balakrishnan and M. Thoma

55

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Ganti Prasada Rao

Piecewise Constant  
Orthogonal Functions  
and Their Application to  
Systems and Control

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To my parents

GANTI VENKATAPPADU

and

GANTI RAJESWARAMMA

with love and regards

### About the author

GANTI PRASADA RAO was born in Seethanagaram, Andhra Pradesh, India, on the 25th August, 1942. He studied at the College of Engineering, Kakinada, and received the B.E. degree in Electrical Engineering from Andhra University, Waltair (India), in 1963 with first class and high honours. He received the M. Tech. (Control Systems Engineering) and Ph. D. Degrees in Electrical Engineering in 1965 and 1969 respectively, both from the Indian Institute of Technology (I.I.T.) Kharagpur.

From July 1969 to October 1971 he was with the Department of Electrical Engineering, PSG College of Technology, Coimbatore, as an Assistant Professor. In October 1971 he joined the Department of Electrical Engineering, I.I.T. Kharagpur as an Assistant Professor and became a Professor in May 1978. From May 1978 to August 1980, he was the Chairman of the Electrical Engineering Curriculum Development Cell at I.I.T. Kharagpur. From October 1975 to July 1976, he was with the Control Systems Center, UMIST, Manchester, England, as a Commonwealth Post doctoral Research Fellow. Presently he is with the Lehrstuhl für Elektrische Steuerung und Regelung, Ruhr-Universität Bochum, West Germany, as a Research Fellow of the Alexander von Humboldt Foundation.

He has research publications in the areas of mathematical instruments, time-varying systems, parametric phenomena, system identification, applications of Walsh and related piecewise constant functions, and fuzzy logic control.

Professor GANTI PRASADA RAO is a Senior Member of the IEEE and a Fellow of the Institution of Engineers (India).

## PREFACE

Walsh functions appeared in mathematical literature sixty years ago. About thirty years later, extensive applications of Walsh functions in communications and signal processing have been suggested. It was in the last decade that their potential as attractive basis functions for signal characterization in control problems became evident. There have been parallel developments with block-pulse functions showing definite advantages in computational aspects.

This book attempts to unify Walsh functions, block-pulse functions, Haar functions etc. into a general class of piecewise constant orthogonal basis functions and presents a comprehensive account of the various applications of these functions in problems such as analysis, optimization and identification of continuous time dynamical systems. The overall treatment is in terms of general piecewise constant orthogonal basis functions although in some particular situations and illustrations, Walsh functions or block-pulse functions are preferred in view of simplicity.

The work of the author with his past students Drs. L. Sivakumar, T. Srinivasan and K.R. Palanisamy in India and the several interesting contributions by Professors C.F. Chen, S.G. Tzafestas, L.S. Shieh, Y.P. Shih and R.R. Mohler and their colleagues abroad, form the core of this book. It is hoped that this book would be of interest to both applied mathematicians and control engineers.

The author is grateful to many of his colleagues and students for their helpful roles in shaping this book. In particular, at IIT Kharagpur, Professor C.N. Kaul (Department of Mathematics) and Professor N. Kesavamurthy gave valuable comments. Professor H. Unbehauen at the Ruhr-Universität Bochum took much interest and encouraged the author in the preparation of the manuscript giving constructive suggestions at several stages. The author thanks Frau H. Hupp, Frau P. Kiesel and Frau E. Schmitt for their skill in transforming an untidy manuscript into a neat typescript. He is grateful to Frau M.-L. Schmücker and Frä. H. Vollbrecht for their excellent draftsmanship.

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Bochum,  
January, 1983.

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