

Lecture Notes in Computer Science

Edited by G. Goos and J. Hartmanis

122

Algorithms in Modern Mathematics and Computer Science

Proceedings, Urgench, Uzbek SSR
September 16–22, 1979

Edited by A.P. Ershov and D.E. Knuth



Springer-Verlag
Berlin Heidelberg New York 1981

Editorial Board

W. Brauer P. Brinch Hansen D. Gries C. Moler G. Seegmüller
J. Stoer N. Wirth

Editors

Andrei P. Ershov
Computing Center
Novosibirsk 630090, USSR

Donald E. Knuth
Dept. of Computer Science
Stanford University
Stanford, CA 94305, USA

AMS Subject Classifications (1980): 01, 03, 68
CR Subject Classifications (1979): 1.2, 2.1, 5.21, 5.25, 5.27

ISBN 3-540-11157-3 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-11157-3 Springer-Verlag New York Heidelberg Berlin

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Printed in Germany

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.
2145/3140-543210

Foreword

SOME TIME AGO a group of mathematicians and computer scientists (including the editors of this volume) thought of making a scientific pilgrimage to the birthplace of al-Khwārizmī, the outstanding ninth-century mathematician who gave his name to the word "algorithm". As his name indicates, al-Khwārizmī came from the Khorezm oasis, a celebrated center of civilization that has given mankind a whole constellation of remarkable philosophers, scientists, and poets. We had learned not only that al-Khwārizmī's famous writings eventually led to our word algorithm, but that the word "algebra" also stems from the title of his major work—thereby accounting for both algebra and algorithms, two of the most important concepts in all of mathematics and computer science. So we felt that a visit to this site would be a special experience for every mathematician, whether he works on abstract or concrete questions, whether he studies philosophical foundations or applications to modern society.

After discussing this idea with a wider circle of scientists, and also consulting with the Academies of Science in the Soviet Union and in the Uzbek S.S.R., our dreams became a reality: An international symposium on the theme *Algorithms in Modern Mathematics and Computer Science* was held during the week October 16–22, 1979, in the city of Urgench, the modern center of the Khorezm region in Uzbekistan. The symposium was organized by the Academy of Sciences of the Uzbek S.S.R., with the support of the Soviet Academy and its Siberian branch.

In order to help prepare for a fruitful pilgrimage, we had written the following letter to potential participants during the planning stages:

It seems to us that this is a splendid opportunity for a truly unique and fruitful conference, an unforgettable experience. Instead of having just another "ordinary" symposium at which we read prepared papers, we prefer to have mutual discussions about fundamental problems of mathematics and computer science. We believe that the desert setting and the atmosphere of history that pervades the conference site will provide us with a special opportunity to take our minds away from the everyday work that fills our lives at home; it should help inspire us to thoughts of a more penetrating, far-sighted, and philosophical nature. Nevertheless, we don't believe that the work of the symposium will succeed if people come completely unprepared, expecting an entirely spontaneous discussion; some serious preliminary study will enable us to work better and to learn better during the time we are together. The attendees will no doubt be specialists in a variety of disciplines with a variety of different modes of thinking, and we certainly don't expect that a sudden unifying consensus of opinion will be reached about the intrinsic nature of algorithms; yet we do feel that the symposium will be an important experience leading to future progress.

It gives us great pleasure now to look back and see that our high hopes for an unforgettable week were indeed fulfilled and surpassed.

The following tentative suggestions for discussion topics, representing problems of common interest about which the participants at the symposium might wish to reach a mutual understanding, were sent out before the meeting:

1. Is there a fundamental distinction between "algebraic" and "algorithmic" methods in mathematics?
2. What is the best way to communicate algorithms between people?

3. What should one do when confronted with a special case of a problem that is algorithmically unsolvable in general?
4. What is the right way to synthesize computer programs containing, say, more than a million instructions?
5. How would the original work of al-Khwārizmī be expressed in modern notation?
6. What sorts of non-algorithmic languages for programming are desirable?
7. Do algorithms provide ideal models for the organization of specific branches of knowledge (e.g., biological sciences, physical sciences, social sciences, music, or mathematics itself)?

Of course, these questions were merely intended to indicate the flavor of the discussions we hoped to incite, rather than to set up strict boundaries about what topics would be treated.

Several of the potential participants added additional comments that helped to set the theme of our meeting. For example:

Is it possible to formulate an appropriate generalization of the Church thesis that would embrace computability in arbitrary object domains and, particularly, computability with probabilistic and nondeterministic devices? [D. Skordev]

Your ideas for a symposium in the Khwarizm region sound very good. Too many researchers work on problems simply because the problems have been proposed as interesting by others. The result is a large number of competent papers leading nowhere. Some reflection on why particular problems are important and what type of advances are possible is much needed. . . . You might add to the list of topics one on mathematical notation. I have often wondered why natural problems so often turn out to be complete for some class. One would expect that the probability of a problem being complete is vanishingly small. There must be something about our notation that forces us into considering only nice problems. [J. Hopcroft]

What are algorithms on real numbers? [N. N. Nepeivoda]

I am interested in various relationships of the concept of algorithm with other mathematical notions: algorithms vs. enumerable sets, algorithms in logic, algorithms and automata and other processes, algorithms and problems with a high degree of complexity, relative computability. [G. S. Tseytin]

I am especially interested in 'How to create an algorithm'. [E. H. Tyugu]

There is a view in which 'algorithmic' and 'algebraic' approaches look almost identical. Maybe the real alternative is 'algorithmic' vs. 'set-theoretical'? Can the concept of algorithm be defined in terms of other standard (say, set-theoretic) mathematical notions or is it essentially independent and primary? [V. A. Uspensky]

What would be a good course on algorithms and logic to be taught at computer science departments? Is it worth while to look for an invariant characterization of computable functions, algorithms, processes of computation? [A. P. Ershov]

It was necessary to limit the size of the symposium to comparatively few delegates in order to keep it from being unwieldy, but we soon realized that the discussions would be stimulating and valuable to a large number of people. Therefore most of the proceedings were subsequently written down, and they are presented here in English. We hope that many readers will now be able to share at least partly in the excitement of that week.

The participants of the symposium are deeply grateful to their Uzbek hosts who showed extraordinary hospitality—what a joy it was for all of us! Special thanks are due to S. Kh. Sirazhdinov, vice president of the Uzbek Academy of Sciences and chairman of the organizing committee; to R. I. Ishchanov, chairman of the executive committee of the Khorezm region; to V. K. Kabulov, director of the Institute of Cybernetics of the Uzbek Academy and co-chairman of the organizing committee; and to K. Š. Babamuradov, division manager of the Institute of Cybernetics and vice-chairman of the organizing committee.

We also are grateful to the many wonderful people we met in the city and in the surrounding countryside, who gave us a warm welcome that we will always remember. Tours were arranged by which we were able to visit historical sites as well as modern schools, farms, and industries. We were impressed that so many people showed great interest in the work of our symposium, following its progress in the newspapers and on television. A public monument in the center of Urgench was dedicated to al-Khwārizmī's memory in a special ceremony when we arrived. It pleases us very much to know that the tradition of al-Khwārizmī lives on in his home territory.

—A. P. Ershov and D. E. Knuth

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