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PREFACE

As already mentioned by Luc Wuytack (in the preface to the proceedings of the conference held in Antwerp 1979, LNM 765) the interest in Padé approximation, convergence acceleration and related subjects is still growing: the conference on "Padé and Rational Approximation, theory and applications" held at the Instituut voor Propedeutische Wiskunde of the University of Amsterdam on October 29-31, 1980, was attended by approximately fifty mathematicians and physicists.

In this publication the reader can find the papers that were presented during the Amsterdam conference. Two of the speakers, however, had their papers accepted for publication elsewhere (A. Sidi & D. Levin: Rational Approximations from the d-Transformation; J. Fleischer & M. Pindor: On an application of the operator Padé Approximants to Quantum Scattering Problems), these talks are not included. Altogether there were thirty lectures: four invited one-hour talks (part I) and twenty six half hour communications (of which 24 appear in part II).

The conference in Amsterdam already was the sixth in a series of conferences on this subject in Europe (Canterbury '72, Toulon '75, Lille '77 and '78, Antwerp '79) and it turned out that the participants would welcome a continuation along this line; some people even offered to organize a conference in 1982 and in 1983.

Finally we would like to express our gratitude to the "Faculteit der Wiskunde en Natuurwetenschappen" of the University of Amsterdam for the financial support and of course, last but not least, we thank the editors of the Lecture Notes and Springer-Verlag for their help and speedy publication of this volume.

Amsterdam, July 29, 1981

M.G. de Bruin

H. van Rossum

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