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Allen Tannenbaum

Invariance and System Theory: Algebraic and Geometric Aspects

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Author

Allen Tannenbaum
Department of Theoretical Mathematics
The Weizmann Institute of Science
Rehovot/Israel

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INTRODUCTION

These lecture notes are based on a series of lectures given by the author at the Mathematical System Theory Institute of the ETH, Zürich, in the Spring of 1980. The purpose of the lectures were two-fold:

(1) To introduce theoretical mathematicians to some of the ideas and methods of system theory, and to attempt to convince them that despite the applied aspects of the field that there are, in system theory, some interesting and deep problems from pure mathematics to be solved and thus for which their techniques could be of great value.

(2) To introduce people working in system theory to the ideas of algebraic geometry, differential geometry, algebraic topology, and invariant theory (but with by far the strongest emphasis on algebraic geometry and geometric invariant theory) which are currently becoming popular in algebraic system theory.

The choice of topics given here was of course dictated by the author's personal tastes and prejudices, and also by the research interests of those working at the ETH in system theory during 1980. Thus even though we believe we have covered a suitably wide range of topics, we make no claims of completeness. As a supplement to these notes, there are now two very nice survey papers in the literature about applications of algebraic geometry in system theory by Byrnes-Falb [19] and Hazewinkel [56].

We now briefly sketch the contents of these notes. Part I is concerned with giving just that part of algebraic geometry which we feel to be most useful in system theory. It should be regarded more as a guide to the literature than anything else, but we hope it will at least acquaint the reader with some of the more important concepts of algebraic geometry.

In Part II, there is a similar sketch of the basic notions of system and control theory. An added feature however in our treatment is an argument due to Deligne [26] concerning the equivalence of the state space and differential equation formulation of certain kinds of systems.

In Part III, we discuss those facts about algebraic groups and geometric invariant theory that we will need in Part IV. In particular, we discuss some basic moduli problems (based on Mumford [104] and [108]) as well as defining the important notions of "quotient" and "geometric quotient".

Perhaps the first non-trivial use of algebraic geometry (and implicitly geometric invariant theory) in system theory is due to R. Kalman who described the quotient space of completely reachable systems of fixed dimension modulo the action of the general linear group as a smooth quasi-projective variety. To do this he essentially generalized the Grassmannian construction. (See Kalman [79] for details as well as Part IV, Section 2 of these notes.) This construction has had many important consequences among which showing that except in the scalar input case, there exist no global algebraic canonical forms (Hazewinkel-Kalman [58] and Part IV, Section 4 of these notes). There has since been quite a large literature about the Kalman construction and its generalizations, e.g., Hazewinkel [54], Byrnes-Gauger [20], Byrnes-Hurt [21]. We will try in Part IV to present these ideas in as simple a form as possible, and specifically as natural constructions coming from Mumford's geometric invariant theory as described in Part III. In point of fact, we will see that the moduli properties (Hazewinkel [53], [54]) and even the existence of the quotient space of completely reachable systems as a smooth quasi-projective variety, should be regarded as generalizations of Mumford's treatment of endomorphisms as given in his Oslo talks [108].

We have remarked already that except in the scalar input case, there exist no global algebraic canonical forms. This still leaves open the question of local canonical forms. Generalizing some very nice work of Arnold [5], in Part V of these notes we will see how to derive local canonical forms not only for completely reachable systems, but also for arbitrary linear time-invariant dynamical systems.

In Part VI we describe the so-called "system realization problem" which concerns the construction of a state space model of a system from its input/output behavior. This will give us an opportunity to describe some of the results about linear systems defined over commutative rings in which a number of techniques from algebraic geometry will be needed. Moreover, we will discuss some recent work of Sontag [139] about realizations of polynomial input/output maps which will give us a first-hand look at some of the properties of dynamical systems which are not necessarily linear.

Part VII is concerned with the geometry of rational transfer functions and since here some algebraic topology is used (the necessary definitions are included), the flavor is of a different type than the preceding parts. We have included this topic since besides its importance in system identification theory, we will see that the results indeed relate to some of the preceding work as well as the blending problem described in Part VIII.

Finally in Part VIII, we come to a topic of great classical and modern importance, namely stabilization through feedback. We describe in some detail the results about state feedback including the famous "pole-shifting theorem" as well as more recent results concerning systems defined over rings. We discuss the work of Youla _et al_ [154] concerning stabilization through dynamic output feedback, and the generalization of this work in what is known as the "blending problem". We include at the end a specific system design.

We would like to thank Professor Rudolf Kalman for inviting us to give these lectures as well as introducing us to the whole subject of algebraic system theory. We also benefitted a great deal from numerous conversations with M. Hazewinkel about algebro-geometric methods in system theory, and H. Kraft about invariant theory (especially the applications in Part IV).

This work was done while the author was a guest at the Forschungsinstitut of the ETH, Zürich. The author spent a splendid two years at this institution, and he wishes to sincerely thank Professor Beno Eckmann for inviting him to come and for his kindness during the author's stay. We also would like to thank the staff for their hospitality. Finally, for the superb job of typing this manuscript, we thank Mrs. Ruby Musrie of the Weizmann Institute of Science, Rehovot, Israel.

NOTATION AND TERMINOLOGY

We have tried to follow the standard notational and terminological practices common to pure mathematics and algebraic system theory. We list here a few of the more commonly used symbols, conventions, and prerequisites for reading the text:

(i) If G is a group acting on a set S, the stabilizer subgroup of $s \in S$ will be denoted by $\mathrm{stab}(s)$, and the orbit of s by $O(s)$.

(ii) For R a commutative ring with unity, $M_{n,m}(R)$ will stand for the set of $n \times m$ matrices with coefficients in R.

(iii) If $A \in M_{n,m}(R)$, then $A^t \in M_{m,n}(R)$ will stand for the transpose of A.

(iv) For R a commutative ring with unity, we use the symbols $GL(n,R)$ and $GL(R^n)$ interchangeably for the group of R-linear automorphisms of R^n.

(v) $\mathbb{R}$: = field of real numbers.

(vi) $\mathbb{C}$: = field of complex numbers.

(vii) $\mathbb{N}$: = set of positive integers.

(viii) We will use several times in the text some standard terms from category theory (for example "contravariant functor", "morphism of functors", etc.). For accounts of all this see [91] or [142].

(ix) The basic definitions needed from algebraic geometry and system theory will of course be given in the text. In certain places we will use some elementary notions from the theory of complex spaces and complex manifolds. Usually we will define all the relevant terms. However for some of the standard terminology which may not be familiar to the reader, everything we use may be found in references [24], [45], [49], [103] and [151] (as well as other places).