

Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

762

D. H. Sattinger

Group Theoretic Methods in Bifurcation Theory

With an



Springer-Verlag
Berlin Heidelberg New York 1979

Author

D. H. Sattinger
School of Mathematics
University of Minnesota
Minneapolis, Minnesota 55455
USA

AMS Subject Classifications (1970): 35JXX, 35KXX, 47H15, 76-XX

ISBN 3-540-09715-5 Springer-Verlag Berlin Heidelberg New York

ISBN 0-387-09715-5 Springer-Verlag New York Heidelberg Berlin

Library of Congress Cataloging in Publication Data

Sattinger, David H

Group theoretic methods in bifurcation theory.

(Lecture notes in mathematics; 762)

Bibliography: p.

Includes index.

1. Differential equations, Partial--Numerical solutions. 2. Bifurcation theory.

3. Representations of groups. I. Title. II. Series: Lecture notes in mathematics (Berlin); 762.

QA3.L28 no. 762 [QA377] 510'.8s [515'.353] 79-23605

ISBN 0-387-09715-5

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks. Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to the publisher, the amount of the fee to be determined by agreement with the publisher.

© by Springer-Verlag Berlin Heidelberg 1979

Printed in Germany

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.

2141/3140-543210

PREFACE

This set of lectures was given in the winter and spring of 1978 while I was on sabbatical at the University of Chicago. I have tried to present the fundamental ideas involved in the combination of group representation theory and bifurcation theory. In addition, there is a chapter by Peter Olver on the derivation of the symmetry group of a differential equation by algebraic methods.

I would like to thank the Mathematics Department of the University of Chicago for their kind hospitality, Peter Olver for his useful remarks during the lectures and his contribution to these notes, and F. Flowers for an excellent job of typing.

My sabbatical was supported by the University of Minnesota, the National Science Foundation (MCS 73-08535) and the U.S. Army Research Office (DA AG 29-77-G-0122), whose support is much appreciated.

David H. Sattinger
June 1978

TABLE OF CONTENTS

I	PHYSICAL EXAMPLES OF BIFURCATION	1
II	MATHEMATICAL PRELIMINARIES	18
III	STABILITY AND BIFURCATION	37
IV	BIFURCATION AT MULTIPLE EIGENVALUES	70
V	ELEMENTS OF GROUP REPRESENTATION THEORY	96
VI	APPLICATIONS	151
VII	APPENDIX: HOW TO FIND THE SYMMETRY GROUP OF A DIFFERENTIAL EQUATION	200
	(by Peter Olver)	
	Subject Index	240