

Hydrodynamic Instabilities and the Transition to Turbulence

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Preface to the Second Edition

In the four years that have elapsed between the first and second editions of this book, much progress has been made in understanding hydrodynamic instabilities and the transition to turbulence. For example, the strange attractors discussed theoretically by Lanford in Chap. 2 have been convincingly observed in experiments on weakly turbulent flows, and several “universal” routes to chaos have been identified in theoretical and experimental studies. Many other noteworthy advances have been made using quite different theoretical methods. For example, the evolution of convection patterns has been studied using two-dimensional model equations.

Brief descriptions of these and other developments, along with numerous added references, are included in this second edition. We hope that the reduced cost of this edition in paperback will make it accessible to many additional scientists and students in the various fields to which it is relevant, especially physics, mathematics, and engineering.

We appreciate the assistance of our contributors, and the support of the National Science Foundation Fluid Mechanics Program.

We dedicate this book to the memory of our colleague and friend, Richard C. DiPrima (9 August 1927–10 September 1984), whose contributions to hydrodynamic stability theory will long be remembered.

Austin and Haverford, February 1985

H. L. Swinney · J. P. Gollub

Preface to the First Edition

Although much of the universe is filled with fluids in turbulent motion, the processes by which turbulence develops are poorly understood. When a fluid is driven away from thermal and mechanical equilibrium, it will often undergo a sequence of *instabilities*, each of which leads to a change in the spatial or temporal structure of the flow. The nature of these instabilities, which sometimes lead to turbulence, is the subject of this volume.

Hydrodynamic instabilities and turbulence have been extensively studied for more than a century, but the research has been primarily concerned with either the first instability that occurs with increasing Reynolds number or with turbulence at very large Reynolds number. The transition from laminar to turbulent flow has until recently been largely beyond the reach of both theory and experiment. This situation has been changed dramatically by the use of computers in laboratory experiments and in numerical analyses of nonlinear systems. While past experiments were primarily photographic or measured time-averaged quantities, recent experiments using computers and modern optical and cryogenic techniques have distinguished between many different dynamical regimes of flows undergoing transition. Numerical studies of nonlinear models have also revealed entirely unexpected results, such as chaotic behavior in a system with only three variables. Another development of great potential importance is the application of new mathematical concepts from the qualitative theory of differential equations, sometimes known as dynamical systems theory, to the transition to turbulence problem. More traditional methods such as bifurcation theory and stability analysis also continue to contribute major new insights.

This book is a collaboration between physicists, mathematicians, and fluid dynamicists, each of whom is a recognized leader in the field. The various chapters include: introductions to the relationship between dynamical systems theory and turbulence (Chaps. 2 and 4); a review of hydrodynamic stability and bifurcation theory (Chap. 3); three case studies – convection, rotating fluids, and shear flows (Chaps. 5–7); a review of the many types of instabilities that occur in geophysics (Chap. 8); and a discussion of instabilities and chaotic behavior in nonhydrodynamic systems (Chap. 9).

Although not all of the book is strictly introductory, the authors have tried to make the majority of it accessible to physicists, mathematicians, engineers, and graduate students who do not have significant background in fluid dynamics

and advanced mathematics. It is our hope that it will provide an introduction to the literature of this rapidly developing field.

We owe special thanks to D. D. Joseph for his encouragement and advice in this endeavor, and to our contributors for their efforts to communicate with clarity to a new audience. We also acknowledge the support of the National Science Foundation.

Austin and Haverford, October 1980

H. L. Swinney · J. P. Gollub

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