

Lecture Notes in Mathematics

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J. A. Green

Polynomial
Representations of GL_n



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Author

James A. Green
Mathematics Institute
University of Warwick
Coventry CV4 7AL
England

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POLYNOMIAL REPRESENTATIONS OF GL_n

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