

The Computer in Optical Research

Methods and Applications

Edited by B. R. Frieden

With Contributions by

R. Barakat W. J. Dallas B. R. Frieden L. Mertz

R. J. Pegis A. K. Rigler

With 92 Figures

Springer-Verlag Berlin Heidelberg New York 1980

Professor *B. Roy Frieden*, PhD

Optical Sciences Center, University of Arizona, Tucson, AZ 85721, USA

ISBN 3-540-10119-5 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-10119-5 Springer-Verlag New York Heidelberg Berlin

Library of Congress Cataloging in Publication Data. Main entry under title: The Computer in optical research methods and application. (Topics in applied physics; v. 41) Bibliography: p. Includes index. I. Optics—Data processing. I. Frieden, B. Roy, 1936—. II. Barakat, Richard, 1931—. QC355.2.C65 535'028'54 80-15930

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Printed in Germany

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Monophoto typesetting, offset printing and bookbinding: Brühlsche Universitätsdruckerei, Giessen
2153/3130-543210

Preface

The ability to measure, and to *compute* with speed and accuracy, are hallmark requirements of the physical or exact sciences. Since its introduction in the late 1950s, the digital computer has furthered the latter ability beyond man's wildest dreams. Today, the computer has uses ranging from mildly useful, to vital, in just about every field of research in optics. Any institution that calls itself a research facility in optics must have a large computer available.

The fields of optics that have particularly profited from the computer's existence are lens and thin film design, image processing and evaluation, and atmospheric optics including remote sensing. The general approaches to problem solving upon a computer fall into three overall areas: *deterministic* or direct calculation methods, as in the evaluation of diffraction integrals; *statistical* methods, as in the Monte Carlo calculation of photographic emulsion characteristics; and *iterative* methods, as in recursive image processing or in optimization problems of lens design. It is remarkable that all these methods stem from the computer's single, only real ability: to add very rapidly.

Of course, as is known to long-term "cybernauts", the machine does have its drawbacks. Computer terminal rooms are notoriously dull places, monopolized by the chatter of the high-speed printer and the visual blight of old Fortran manuals. Worse yet, the computer is the ultimate in harsh taskmasters, requiring a perfect program, with its own, sometimes unpredictable meaning of the word "perfect". Computer programmers often suffer the effects of years of interaction with a rigid and demanding machine. Nevertheless, computer based research has some definite advantages, a not insignificant one being *non-reliance* upon the purchase, delivery, and correct operation of expensive and complicated laboratory equipment. Many a research project and Ph. D. dissertation have foundered on the long time delays caused by these problems.

The aim of this volume is to bring together as wide and representative a scope of problem-solving techniques as can fit within the confines of a modest-sized text. Applications of the techniques to important problems in optics have been emphasized as well. We hope, in this way, to introduce use of "the machine" to newcomers in optics, and to encourage its wider use among even experienced cybernauts.

The following general areas of research techniques are covered. Chapter 1 is an overall survey of computer developments in optics, somewhat from a historical viewpoint, and with a survey of the other chapters. Chapter 2 treats modern methods of computing diffraction integrals. Chapter 3 introduces and

develops probabilistic and statistical methods of analysis. Chapter 4 covers optimization techniques, and especially as they apply to lens and thin film design. Chapter 5 offers an introduction to computer uses in optical astronomy. And Chapter 6 surveys the field of computer-generated holograms.

I would like to thank Dr. Robert E. Hopkins of the University of Rochester for first introducing me to computer based research, and guiding my early career when it so acutely needed guidance.

I would also like to thank Dr. Helmut Lotsch for first suggesting the need for this kind of text. Many thanks are due the authors of the different chapters for their kindness in agreeing to contribute to the book, for their extreme kindness in actually carrying through with the promise, and finally, for their perseverance in meeting the various deadlines and revising and updating where these were necessary.

Tucson, Arizona
August, 1980

B. Roy Frieden

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Contributors

Barakat, Richard

Aiken Computation Laboratory, Division of Applied Sciences,
Harvard University, Cambridge, MA 02138, USA

Dallas, William John

Philips GmbH, Forschungslaboratorium Hamburg, Vogt-Kölln-Str. 30,
D-2000 Hamburg 54

Frieden, B. Roy

Optical Sciences Center, University of Arizona,
Tucson, AZ 85721, USA

Mertz, Lawrence

Lockheed Palo Alto Research Laboratories, 3251 Hanover Street,
Palo Alto, CA 94304, USA

Pegis, Richard John

34 Argyle Street, Rochester, NY 14607, USA

Rigler, A. Kellam

Mathematics Computer Science Building, University of Missouri – Rolla,
Rolla, MO 65401, USA