

# Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

507

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Michael Reed

Abstract Non Linear  
Wave Equations

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Springer-Verlag  
Berlin · Heidelberg · New York 1976

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### Library of Congress Cataloging in Publication Data

Reed, Michael.

Abstract non-linear wave equations.

(Lecture notes in mathematics ; 507)

Based on lectures delivered at the Zentrum  
für interdisziplinäre Forschung in 1975.

Bibliography: p.

I. Wave equation. I. Title. II. Series:

Lecture notes in mathematics (Berlin) ; 507.

QA3.L28 no. 507 [QC174.26.W3] 510'.8s [530.1'24]  
76-2551

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AMS Subject Classifications (1970): 35L60, 47H15

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ISBN 3-540-07617-4 Springer-Verlag Berlin · Heidelberg · New York  
ISBN 0-387-07617-4 Springer-Verlag New York · Heidelberg · Berlin

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Printed in Germany

Printing and binding: Beltz, Offsetdruck, Hemsbach/Bergstr.

## Preface

These notes cover a set of eighteen lectures delivered at the Zentrum für interdisziplinäre Forschung of the University of Bielefeld in the summer of 1975 as part of the year long project "Mathematical Problems of Quantum Dynamics". It is a pleasure to thank the Zentrum for the opportunity to give these lectures and the Physics faculty of the University of Bielefeld for their warmth and hospitality. Three people deserve special thanks: L. Streit, for extending the invitation, C. Pfister, for help in the preparation of the manuscript, and M. Kämper for her excellent typing.

Mike Reed

Bielefeld, August, 1975

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