

# Lecture Notes in Mathematics

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# Lecture Notes in Mathematics

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## Numerical Analysis

Proceedings of the Dundee Conference  
on Numerical Analysis, 1975

Edited by G. A. Watson

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## Foreword

For the 4 days July 1-4, 1975, around 200 people attended the 6th biennial conference on numerical analysis at the University of Dundee, Scotland. Previous conferences have in the main been concerned with specific subject areas, such as the numerical solution of differential equations. This year, however, it was decided to broaden the scope of the meeting to encompass the whole of numerical analysis, while maintaining a bias towards the more practical side of the subject.

Invitations to present talks were accepted by 16 eminent numerical analysts, representative of a variety of fields of activity, and their papers appear in these notes. In addition to the invited papers, short contributions were solicited, and 45 of these were presented at the conference in parallel sessions. A list of these papers is given, together with the addresses of the authors (correct at the time of the conference). I would like to thank all speakers, including the after dinner speaker at the conference dinner, Mr A R Curtis, all chairmen and participants for their contributions.

It is not always realised that the Dundee numerical analysis conferences are firstly, financially self-supporting, and secondly, organised entirely from within the Department of Mathematics. As on so many previous occasions, the organisation of the conference was in the very capable hands of Dr J Ll Morris, assisted by various other members of the Mathematics Department. Particularly in view of the recent departure of Dr Morris from Dundee to the University of Waterloo, I would like to take this opportunity to pay tribute to the very considerable contribution he has made to the continued success of the numerical analysis conferences in Dundee.

The typing of the various documents associated with the conference and some of the typing in this volume has been done by secretaries in the Mathematics Department, in particular Miss R Dudgeon; this work is gratefully acknowledged.

G A Watson

Dundee, September 1975.

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INVITED SPEAKERS

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Salt Lake City, Utah 84112, U.S.A.

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Wilhelms-Universität, D-44 Münster, Germany.

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Harwell, Didcot, Oxfordshire, England.

J K Reid Computer Science and Systems Division, A.E.R.E.  
Harwell, Didcot, Oxfordshire, England.

H J Stetter Institut für Numerische Mathematik, Technische  
Hochschule Wien, A-1040 Wien, Gusshausstr, 27-29  
Austria.

### Submitted Papers

A Z Aktas and H Öncül: Computer Science Dept., Middle East Technical University, Ankara, Turkey.

Some numerical methods for nonlinear boundary value problems in O.D.E's.

R Alt: Institut de Programmation, Faculte des Science, Universite de Paris, Tour 55 - 11 Quai Saint-Bernard, Paris 5.

Evaluation of the numerical error committed in the floating point computation of a scalar product.

D E Amos: Numerical Division, Sandia Laboratories, Albuquerque, New Mexico.

Computation of I and J Bessel functions for real, non-negative orders and arguments.

E Ball\* and R A Sack<sup>+</sup>: \*Dept of Electrical Engineering and <sup>+</sup>Dept of Mathematics, University of Salford, Salford, England.

Numerical quadrature of line integrals.

K E Barrett: Mathematics Dept, Lanchester Polytechnic, Coventry, England.

Applications and extension of a variational principle for the stream function-vorticity formulation of the Navier-Stokes equations incorporating no slip conditions.

C Brezinski: University of Lille, France.

Computation of Padé approximants.

C Carter: Trent University, Peterborough, Ontario, Canada.

Evaluation of the greatest eigenvalue of an irreducible non-negative matrix.

F H Chipman: Mathematics Dept, Acadia University, Wolfville, N.S., Canada.

Implicit A-stable R-K methods with parameters.

M G Cox: Division of Numerical Analysis and Computing, National Physical Laboratory, Teddington, Middlesex, England.

The numerical evaluation of a spline from its B-spline representation.

L M Delves and J M Watt: Department of Computational and Statistical Science, University of Liverpool, England.

A proposal for a Gauss quadrature library package.

J C Eilbeck and G R McGuire: Mathematics Dept, Heriot-Watt University, Riccarton, Currie, Midlothian, Scotland.

Finite difference methods for the solution of the regularized longwave equation.

N T S Evans\* and A R Gourlay<sup>+</sup>: \*MRC Cyclotron Unit, Hammersmith Hospital, London, England and <sup>+</sup>IBM UK Scientific Centre, Peterlee, Co Durham, England.

The solution of a diffusion problem concerned with oxygen metabolism in tissues.

R Fletcher and T L Freeman: Mathematics Dept, The University, Dundee, Scotland.

A modified Newton method for minimization.

W Forster: Mathematics Dept, The University, Southampton, England.

The structure of computational methods: A note on consistency, convergence, and stability.

R Frank: Institut für Numerische Mathematik, Technische Hochschule Wien, A-1040 Wien, Gusshausstr, 27-29 Austria.

The method of Iterated Defect-Correction.

T L Freeman, D F Griffiths and A R Mitchell: Mathematics Dept, The University, Dundee, Scotland.

Complementary variational principles and the finite element method.

- J H Freilich and E L Ortiz: Mathematics Dept, Imperial College, London University, England.  
Tau method approximation to the solution of 2nd order linear differential equations.
- I Gargantini: University of Western Ontario, London, Ontario, Canada.  
Parallel Laguerre iterations: The complex case.
- E Hairer: Mathematics Dept, Université de Genève, Switzerland.  
Equations of condition for Nystroem methods.
- P J Hartley: Mathematics Dept, Lanchester Polytechnic, Priory Street, Coventry CV1 5FB, England.  
Some tensor product, hypersurface fitting methods.
- J G Hayes: National Physical Laboratory, Teddington, England.  
Bicubic splines with curved knot-lines.
- T R Hopkins\* and R Wait<sup>†</sup>: \*Computing Laboratory, University of Kent, Canterbury, England and <sup>†</sup>Dept of Computational and Statistical Science, University of Liverpool, Liverpool, England.  
A comparison of numerical methods for the solution of quasi-linear P.D.E's.
- E S Jones: Dept of Computing Science, The University of Glasgow, Glasgow, Scotland.  
Quasi-Newton methods for non-linear equations: Line search criteria and a new update.
- R B Kelman and J T Simpson: Dept of Computer Science, Colorado State University, Ft Collins, Colorado 80523, U.S.A.  
Algorithms for solving dual trigonometric series.
- F M Larkin: Dept of Computing and Information Science, Queen's University, Kingston, Ontario, Canada.  
A note on the stability of Ritz-type discretizations of certain parabolic equations.
- T Lyche: Mathematics Dept, University of Oslo, Oslo 3, Norway.  
Asymptotic expansions and error bounds for cubic smoothing splines.
- D Meek: Mathematics Dept, Brunel University, Uxbridge, Middlesex, England.  
Toeplitz matrices with positive inverses.
- C A Micchelli and A Pinkus: IBM, Research Division, Yorktown Heights, NY 10598, U.S.A.  
On n-widths in  $L^\infty$ .
- H D Mittelmann: Fachbereich Mathematik, Der Technischen Hochschule Darmstadt, 61 Darmstadt, Kantplatz 1, Germany.  
On pointwise estimates for a finite element solution of nonlinear boundary value problems.
- E Moore: Memorial University of Newfoundland, St John's, Newfoundland, Canada.  
Curve fitting using integral equations.
- M Neumann: Israel Institute of Technology, Haifa, Israel.  
Subproper splitting for rectangular matrices.
- M A Noor: Mathematics Dept, Brunel University, Uxbridge, England.  
Error bounds for the approximation of variational inequalities.
- J Oliver: Computer Centre, University of Essex, Colchester, England.  
A curiosity of low-order explicit Runge-Kutta methods.



I E Over, Jr: Lowell University, Lowell, Mass., U.S.A.  
A modern course for training student engineers.

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51 Aachen, Templegraben 64, W Germany.  
On a system of hyperbolic variational inequalities.

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Bristol, England and <sup>+</sup>Mathematics Dept, University of Aberdeen, Aberdeen, Scotland.  
On the numerical solution of multiparameter eigenvalue problems in ordinary differ-  
ential equations.

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On Galerkin methods for stabilized Navier Stokes problems.

E Spedicato: CISE, PO Box 3986, 20100 Milan, Italy.  
A three parameter class of quasi-Newton algorithms derived from invariancy to non-  
linear scaling.

P Spellucci: Mathematics Dept, University of Mainz, Germany.  
A modification of Wittmeyers method.

W J Stewart: Laboratoire d'Informatique, Université de Rennes, France 35000.  
Markov modelling using simultaneous iteration.

P G Thomsen and Z Zlatev: Institute for Numerical Analysis, Technical University  
of Denmark.  
A two-parameter family of PECE methods and their application in a variable order,  
variable stepsize package.

G Varga: Computer and Automation Institute, Hungarian Academy of Sciences, Budapest,  
Hungary.  
A relaxation method for computation of the generalized inverse of matrices.

M van Veldhuizen: Wiskundig Seminarium, Vrije Universiteit, Amsterdam, Netherlands.  
A projection method for a singular problem.

J G Verwer: Mathematisch Centrum, Amsterdam, Netherlands.  
S-stability for generalized Runge-Kutta methods.

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Unconstrained optimization by approximation of the gradient path.