

Lecture Notes in Mathematics

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Measures on
Topological Semigroups:
Convolution Products and
Random Walks



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Preface

This monograph is an outgrowth of the lecture notes of a series of lectures given by the first author in the Indian Statistical Institute during the fall of 1973. These notes supplement in many ways the material presented in the book "Probabilities on Algebraic Structures" by Ulf Grenander and the material that appears in Chapters IV and V of the book "Markov Processes: Structure and Asymptotic Behavior" by M. Rosenblatt. Like most mathematicians who have worked in this area, we owe much to these two mathematicians. We also gratefully acknowledge a number of stimulating conversations with Prof. M. Rosenblatt when he was invited to speak in the Wayne State University Semigroup Symposium in 1968 and when he was invited to give a series of colloquium talks at the University of South Florida in early 1973.

Our primary objective in these notes is to provide the reader with a brief, but somewhat complete account of the theory of probability and measure on topological semigroups in the context of the following problems: (i) the characterization of the idempotent and r^* -invariant probability measures on locally compact Hausdorff topological semigroups (ii) the limit behavior of the averaged and unaveraged sequence of convolution iterates of probability measures on different topological semigroups, and also on semigroups of stochastic matrices (iii) almost sure convergence of products of independent random variables taking values in a completely simple semigroup and (iv) the recurrence behavior of one-sided and two-sided random walks induced by a probability measure on a compact Hausdorff or locally compact Hausdorff completely simple topological semigroup. Thus our notes cover only certain aspects of probability theory on semigroups while leaving out many other interesting aspects such as the study

of infinitely divisible probabilities on groups and semigroups and a discussion of the embedding problem for such measures. Other interesting subjects which we have not even touched include the study of potential theory for recurrent random walks initiated by Spitzer and later studied by Kesten, Ornstein, Port and Stone, Brunel and Revuz, and others. The main reason for these omissions is that these areas of study, while highly explored in the context of groups, have been somewhat overlooked in the general framework of topological semigroups.

We hope that the reader will find the results and the methods that are developed in these notes useful in many different contexts. We feel that these notes can be covered during a one-semester seminar meeting once a week for two hours in a typical American university.

We express our deep appreciation to Professor K. H. Hofmann of Tulane University and Professors A.T. Bharucha-Reid and T. C. Sun of Wayne State University. We have learned a great deal from them on semigroups, measures and probability, through occasional correspondence and actual collaboration .

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