Interplanetary Mission Analysis and Design

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Preface

One of the enduring legacies of the twentieth century will be the advent of space travel. This achievement has changed the way we think about our presence in the Universe and now offers the possibility to explore beyond our own world. Nobody experiencing the Apollo missions of the 1960s and 70s could be unaffected by the magnitude of those achievements. Now, nearly forty years later, space travel is almost commonplace, although man's personal presence in space is still limited.

The pioneering edge of space exploration now lies beyond Earth and its moon, as numerous spacecraft over the last thirty years have visited most of the planets of the Solar System. Mars remains the focus of many such missions and is likely to be the first planet that man visits personally, not just through robotic craft. Perhaps the most fascinating interplanetary missions to date have been the Voyagers. Launched in the late 1970s, these spacecraft were flung out of the Solar System after flying by the outer planets. They have now passed far beyond Pluto as they leave the Sun's domain behind.

These technical achievements have inspired numerous science fiction stories, which in turn have themselves perhaps influenced the drive for new space missions and exploration. Although 'warp drives' remain for the present in the realm of science fiction, more adventurous missions to explore our Solar System are being planned. These include an initiative to place a man on Mars and also for a detailed robotic exploration of Jupiter and its family of moons. This later system has already been inspected by the Galileo spacecraft and revealed a fascinating 'micro solar system' with a rich variety of features. Also, both current (Messenger) and planned future missions (Bepi–Colombo) to Mercury will undertake the difficult route to the innermost planet of the Solar System. In addition to the planets, the minor bodies of the Solar System are being explored. A challenging example is ESA's Rosetta mission, following a complex route to achieve a rendez-vous with a comet.

MISSION ANALYSIS AND DESIGN

This diverse range of missions require numerous techniques for their analysis and design. These aspects will be considered in this book, including the key issues of escaping from a planet, interplanetary transfer, and capture at a target planet. Certain 'classical' methods for the design of such trajectories have been employed for many years. As missions became more demanding, then new techniques were developed to enable more efficient designs to be realised. These allow the efficient utilisation of newly evolving propulsion technologies. This theme is continuing, as both new mathematical and computational ideas are considered for the solution of these problems.

The objective of this book is to describe a selection of techniques that may be applied to the analysis and design of interplanetary missions. The focus is on methods that enable the efficient solution of the problems considered. Details of the methods are given. However, this text is not intended as a reference on astrodynamics. Summaries of key derivations are included.

The terms 'mission analysis' and 'mission design' can have several meanings. The one taken here is that of the analysis and design of spacecraft transfers. Therefore, the focus is on techniques in orbital mechanics and trajectory optimisation that may be applied to the objective. The aspects of mission analysis particularly relevant to interplanetary missions are considered here.

This book is divided into five major chapters. The first chapter focuses on 'conventional' analysis and design techniques for interplanetary missions. This includes the basic ideas of Hohmann transfers, the solution of Lambert's problem and the fundamentals of planetary escape and capture. These basic ideas of interplanetary transfers are then extended to consider return missions to the planets. The issue of escape from Earth is also considered in more detail, in the context of the efficient utilisation of launch vehicle capabilities. This subject is also further expanded in Appendix 4.

The second chapter briefly considers aspects of spacecraft propulsion systems. These systems are a fundamental factor in the nature of interplanetary mission designs and therefore warrant some consideration in a book such as this.

The third chapter focuses on optimisation. In particular, methods for obtaining solutions to local optimisation problems are considered. These generally require gradient evaluations and are often called 'gradient based' methods. These methods are essential for the efficient design of interplanetary missions. As more complex propulsion system types are considered, so the complexity of the optimisation problems increases. Many developments are taking place in this area. Only gradient-based methods are considered here. However, it should be noted that alternative techniques such as evolutionary computing offer very interesting prospects for the identification of globally optimal solutions. In addition to trajectory optimisation problems, the nature of spacecraft optimisation is discussed, where the design of the propulsion installation may be optimised together with the transfer trajectory.

The fourth chapter considers a range of 'special' methods that may be employed for mission analysis and design. These methods allow the planning of more efficient transfers. A consequence of the improved efficiency is the increased complexity of the transfer routes. Some of these methods take advantage of interesting features of astrodynamics. A good example of this is the phenomenon of gravitational escape or capture at a planet, which has been observed for comets in the Solar System. Consequently, this chapter contains an outline description of the three-body problem and mission designs that can utilise three-body effects. Many of the techniques considered in this chapter allow the efficient utilisation of advanced propulsion system concepts. This is particularly true for low-thrust systems. The effect of low thrust systems on orbit evolution is considered via the application of perturbation equations. Considerable attention is paid to 'gravity assist'. This technique allows the utilisation of combined gravity fields to enable a spacecraft to significantly modify its orbital energy, without the need for manoeuvre.

The final chapter describes a series of mission examples that utilise the methods described in the previous sections. These include missions using gravity assist, low-thrust propulsion, gravitational escape and capture. Many of the examples are generic, in that they consider typical transfers between planets. However, certain examples are relevant to actual missions, either past, current, or future.

The appendices describe the basics of orbital mechanics, orbital reference frames, and also the properties of the planets. The data is intended as a source of reference for the material in the book.

A CD is included to give some examples of interplanetary missions. A simulation tool is included. This is used to generate animated sequences that show interplanetary transfers. The missions illustrated include both transfers to the inner and outer planets. Instructions for use are contained on the CD. The software runs on Windows based PC systems.

Although every effort has been made to eliminate mathematical and factual errors in the material in this book, complete accuracy cannot be guaranteed. Please send any errors found and suggested corrections to the author.

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Nomenclature

COMMONLY USED TERMS

The following lists a series of acronyms, mathematical symbols, and phrases frequently used in this text. This is not a comprehensive list and details of further terms used are given in the appropriate sections.

COMMON ACRONYMS

ΔV	Delta-V
EGA	Earth Gravity Assist
GA	Gravity Assist
Isp	Specific Impulse
JGA	Jupiter Gravity Assist
LGA	Lunar Gravity assist
MGA	Mercury Gravity Assist
NEP	Nuclear Electric Propulsion
SEP	Solar Electric Propulsion
SGA	Saturn Gravity Assist
VGA	Venus Gravity Assist
V_∞	Excess hyperbolic speed

GRAVITY ASSIST SEQUENCE LABELLING

E	Earth
J	Jupiter

L	Moon (Earth's)
M	Mercury
S	Saturn
V	Venus

Example: V–E–E = Venus, Earth, Earth gravity assist sequence

COMMON SUBSCRIPTS TO MATHEMATICAL TERMS

C	Denotes term relative to central body
I	Denotes a vector expressed in an inertially oriented frame
p	Denotes term relative to major body

COMMON MATHEMATICAL SYMBOLS

a	Semi-major axis of ellipse
α	Velocity vector deflection angle (in gravity assist description) Thrust vector azimuth angle in low-thrust vector modelling
A_X^Y	Transformation matrix from Y to X frame
b	Semi-minor axis of ellipse
β	Angle describing the location of the intersection of the approaching asymptotic excess hyperbolic velocity vector with the B plane (in gravity assist description) Thrust vector elevation angle in low-thrust vector modelling
e	Eccentricity
C	Jacobi constant (in three-body problem) Constraint (in optimisation problems)
d	Distance
ΔV	Delta-V
E	Eccentric anomaly (in ephemeris elements) Energy (usually per unit mass)
F	Hyperbolic anomaly
Γ	Flight path angle
h	Angular momentum (usually per unit mass)
i	Inclination
J	Objective function
λ	Lagrange multiplier (in non-linear programming methods) Longitude (in orbit kinematics)
m	Mass
M	Mean anomaly
μ	Gravitational parameter (in gravitational acceleration terms) Mass ratio (in three-body problem)
p	Orbit semi-latus rectum

P	Adjoint vector (in indirect optimisation)
r	Spacecraft position (may be used as vector or magnitude)
r_{planet}	Major body position (may be used as vector or magnitude)
t	Elapsed time
T	Thrust (often per unit mass)
τ	Orbit period
θ	True anomaly (in ephemeris elements)
	Angle between approaching relative velocity vector and major body velocity vector in gravity assist
u	Control vector
U	Potential
V	Spacecraft velocity (may be used as vector or magnitude)
V_{planet}	Major body velocity (may be used as vector or magnitude)
ω	Angular velocity (of major body) (may be used as vector or magnitude)
Ω	Right ascension of ascending node
X	State vector
V_∞	Excess hyperbolic speed
V_{rel}	Velocity relative to major body (used in two-body approximation)

NOTATION

$\sum_i^N$	Summation abbreviated form of $\sum_{i=1}^{i=N}$

COMMONLY USED TERMINOLOGY

Term	*Description*
Aerobraking	The process of utilisation of a pericentre passage (or multiple passages) through a major body's atmosphere with the objective of reducing the apocentre.
Aerocapture	The process of utilisation of a pericentre passage through a major body's atmosphere with the objective of reducing the energy of an approaching spacecraft in an initially unbound orbit to reach a bound orbit.
Approach plane	In a gravity assist, the plane containing the velocity of the major body and the asymptotic excess hyperbolic velocity vector.
Asymptotic excess hyperbolic velocity vector	The velocity vector whose magnitude is equal to the excess hyperbolic speed and direction defined by the asymptotes of the hyperbola when either approaching or departing from a major body.
B plane	The plane normal to an approaching asymptotic excess hyperbolic velocity vector and passing through the centre of the major body.

Central body	The primary body with the dominant gravity field in a system. Examples are the Sun for transfers in the Solar System, or Jupiter for transfers to the Jovian moons.
ΔV or Delta-V	The speed change effected by propulsive force to implement a change in orbit. Includes the inefficiency arising from non-impulsive manoeuvres. Implemented by a spacecraft propulsion system or environmental perturbation to the orbit.
Direct optimisation method	Method to determine the optimal control solution that minimises or maximises an objective using the approximation of a parameterisation of a continuously variable control function.
Escape orbit	An unbound orbit (energy > 0), resulting in a hyperbola.
Excess hyperbolic vector	The speed remaining at infinite distance (when in a hyperbolic orbit). In practice, approximated by the speed after leaving the sphere of influence.
Fly-by	A passage close to a major body during the course of a trajectory about a central body.
Gravity assist	The process of utilisation of the gravity field of a major body, during the course of a trajectory about the central body, with the result of energy and angular momentum modification with respect to the central body.
Gravitational escape (or capture) (or gravity assisted escape or capture)	The process of achieving an escape orbit from an initial bound orbit under the influence of the gravity fields of a major and central body. (Reverse for capture.)
Halo orbit	An 'orbit' about a collinear Lagrange libration point whose periods in and out of ecliptic are equal.
Hohmann transfer	A two-impulse transfer over half of an ellipse. Optimal transfer between coplanar, circular orbits.
Indirect optimisation method	Method to determine the optimal control solution that minimises or maximises an objective using variational calculus methods. Pontryagin's maximum (or minimum) principle is employed.
Invariant manifold	A constrained trajectory in phase space describing both the position and velocity vectors. Used in the context of the three-body problem to describe the evolution of a spacecraft trajectory under the influence of combined gravity fields (and neglecting non-gravitational perturbations).
Keplerian orbit	An orbit that is the solution of the two-body problem. Solutions are conic sections: ellipse, parabola or hyperbola.

Lagrange libration point	A point of equilibrium in the three-body problem, considering the gravity fields of two bodies and the centrigugal force term. Five points exist, lying in the orbit plane of the major body about the central body. Three lie along the line passing through the central to major body.
Lambert's problem	The problem of finding an orbit connecting two positions over a particular time interval.
Lissajous orbit	An 'orbit' about a collinear Lagrange libration point whose periods in and out of ecliptic are unequal.
Low thrust	A propulsive force producing a small resultant acceleration on the spacecraft ($\ll 1$ g). Examples are electric propulsion systems and solar sails.
Major body	The secondary body with the locally dominant gravity field in a particular mission phase, such as a fly-by. Examples are a planet encountered during the course of a trajectory about the Sun or a moon during the course of a trajectory about a planet.
Non-gravitational perturbation	A small acceleration term (compared with the gravity field of the central body) applied to an orbiting object, arising from non-gravitational sources (e.g., low-thrust propulsion, solar radiation pressure, atmospheric drag).
Non-linear programming	A iterative method to determine the solution of an optimisation problem by using gradient information regarding the objective and constraints.
Patch conic	The method of approximating the motion of a spacecraft under the influence of the gravity fields of a central and major body by using a sequence of conic sections (each of which is a solution to a two-body problem).
Resonant orbit	An orbit whose period may be expressed as an integer ratio with the period of the orbit of the major body to which it relates.
Secular perturbation	A perturbing term that effects a net orbit change after a complete revolution. When considered over multiple revolutions the mean ephemeris of the orbit undergoes a progressive evolution.
Specific impulse	Impulse delivered per unit mass of propellant.
Sphere of influence	Locus of points around a major body obtained by considering the perturbing gravitational acceleration of the central body compared with the gravitational acceleration for the central body.
Synodic period	Repeat period when considering the re-occurrence of relative longitudes of two major bodies. Assumes coplanar, circular orbits.

Three-body problem	The problem of motion of a spacecraft under the combined influence of the gravity fields of two massive bodies.
Two-body problem	The problem of motion of a spacecraft under the influence of the gravity field of a single massive body.
Weak stability boundary	Term sometimes used to describe a region under the influence of combined gravity fields (of a major and central body).

AXES SYSTEMS USED IN ILLUSTRATIONS

The figures illustrating transfer trajectories are often drawn over a reference grid. The scaling of the grid is detailed for each plot. It is generally 1 AU from centre to edge with a sub-grid size of 0.1 AU.

The grid lies in the ecliptic and the conventional X axis direction (i.e., to the first point in Aries) is in a vertical direction (i.e., to the top of the page). Details of axes and reference systems are given in Appendix 2.