Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

Series: Department of Mathematics, University of Maryland, College Park

Adviser: L. Greenberg

478

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Univalent Functions - Selected Topics

Springer-Verlag

Berlin · Heidelberg · New York 1975

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Library of Congress Cataloging in Publication Data

Schober, Glenn, 1938-
Univalent functions--selected topics.

(Lecture notes in mathematics ; 478)
Bibliography: p.
Includes index.
1. Univalent functions. I. Title. II. Series.
QA3.L28 vol. 478 [QA331] 510'.8s [515'.253]
ISBN 0-387-07391-4 75-23099

AMS Subject Classifications (1970): 30A32, 30A36, 30A38, 30A40, 30A60

ISBN 3-540-07391-4 Springer-Verlag Berlin · Heidelberg · New York
ISBN 0-387-07391-4 Springer-Verlag New York · Heidelberg · Berlin

Offsetdruck: Julius Beltz, Hemsbach/Bergstr.

PREFACE

These notes are from lectures given by the author in 1973-74 at the University of Maryland, during its special year in complex analysis. They are an attempt to bring together some basic ideas, some new results, and some old results from a new point of view, in the theory of univalent functions.

There are really two points of view that are used and intertwined in these notes. The first is to utilize a linear space framework to study sets of univalent functions as they are situated in a space of analytic functions. For example, in Chapter 7 we are interested in compactness of families of univalent functions that lie in the intersection of two hyperplanes, and in Chapter 8 we are interested in their geometry in the sense of convexity theory.

In the same spirit, we consider in Chapter 2 many of the special families of univalent functions and determine the extreme points of their closed convex hulls. This point of view seems to simplify and unify the study of their properties, for example, in solving linear extremal problems. In keeping with this point of view we give in Chapter 1 a derivation of the Herglotz representation based on Choquet's theorem. In this case the route is less elementary, but it serves to establish our point of view.

The second point of view is to study extremal problems using variational considerations. In the absence of a structural formula for a class of functions, variational methods are a very powerful tool. In Appendix C we include the boundary variation from the fundamental work of M. Schiffer, and in Chapters 10 and 11 we apply it to solve some accessible problems and to give geometric properties of solutions to others.

Variational considerations can also be used to study quasiconformal mappings. Our treatment of quasiconformal mappings is not thorough. However, in Chapter 13 we give a very general variational procedure for families of quasiconformal mappings. Some applications are given in Chapter 14. At present, this appears to be a rapidly developing area.

A number of other topics that seem to fit in are included, e.g., the affirmative solution of the Pólya-Schoenberg conjecture, representation of continuous linear functionals, Faber polynomials, properties of quasiconformal mappings, and quasiconformal extensions of univalent functions.

One final comment about the structure of these notes: There are two kinds of problems in the text, those called exercises and those called problems. There is a distinction; namely, the author knows how to solve only the exercises. This comment was delayed until now in the hope that the reader might overlook it, and might go on to solve some of the problems.

Finally, the author wishes to acknowledge the work of Mr. Tom Whitehurst, who proofread these pages, and of Miss Julie Palmer and Mrs. Karen Barker, who typed them.

Bloomington, Indiana — Glenn Schober

December 1974

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