

Lecture Notes in Mathematics

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Computational Problems,
Methods, and Results
in Algebraic Number Theory



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Preface

In this report an attempt is being made to briefly survey, with special emphasis on the computer-oriented view point, numerical investigations centering on Algebraic Number Theory.

The report was initiated while the author attended the Atlas Symposium No. 2 on 'Computers in Number Theory' that took place at Oxford, England, August 18-23, 1969. It was written mainly during the author's participation in the Algebra Year 1969/70 at the University of California at Los Angeles. The author wishes to express his thanks to the Atlas Computer Laboratory at Chilton, Didcot, and to the Department of Mathematics of the University of California at Los Angeles for inviting him to these meetings.

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